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Effects of Insertion Devices on Beam Dynamics of Swiss Light Source

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Table of Contents

1. Introduction :

The SLS is a low emittance and 2.4 GeV energy storage ring with TBA lattice configuration. It has three long (11m), three medium (7m) and two short (4m) straight sections with zero dispersion. The four of these, a long (9L), a medium(11M) and two short straight sections (4S,6S) are used to install the various types of Insertion Devices (IDs) to obtain the photon beams in energy range of 10 eV to 40 keV. The two electromagnetc elliptic undulators EEW (2xUE212) and two helical undulators of Apple II/ Sasaki-type (2xUE56) will be installed in 9L and 11M respectively and operated in double undulator. The two planar IDs, wiggler W61 and in-vacuum undulator U24 will be installed in straight sections 4S and 6S respectively. Later on, U24 will be replaced by another in-vacuum planar undulator U17. The elliptical twin-undulators UE212 will produce two parallel photon beams with opposite linear and circular polarization by producing parallel offset of 1mm of electrom beam in two undulators. The twin-undulators UE56 can be tuned to produce the photon beams of any degree of polarization from circular to linear. The variable and different polarization is achieved by producing variable and different field distributions in two undulators. The electron beam will have parallel offset of 1mm in this case also.

The IDs perturb the linear and non-linear beam dynamics of the storage ring. It can limit the dynamic aperture as well as tune space available for machine operation. Thus can have severe effects on injection efficiency and beam life time. Therefore, it is very important to understand the beam dynamics theoretically as well as to investigate numerically for optical performance of storage ring. The numerical studies can reveal if any re-tuning of machine optics is required locally, globally or both. The detailed theoretical and numerical investigations have been carried out to understand the beam dynamics with various types of IDs planned at SLS and presented in this report.

The various mathematical and simulation models developed at various labs are studied in order to choose the appropriate models for simulation studies of EEW and Apple II devices. The simulation model developed by G. Wüestefeld et al. at BESSY-II and L. Tosi et al. of ELETTRA for simulation studies for Apple II and EEW devices and implemented in computer codes BETA and RACETRACK respectively are discussed briefly. The consideration of effects of End-Pole design in a simulation model is important and discussed here.

2. Theoretical Aspects :

The insertion devices (IDs) are special magnetic devices consisting of sequence of dipoles with alternative polarities and arranged so that there is no net deflection and displacement of the beam when it passes them. But they cause the perturbation of a motion of an electron in the storage ring due to linear and non-linear field perturbations introduced by them. The distortions of linear optics are generated by the edge focusing of magnetic poles. It destroys the super-periodicity of the lattice. The presence of highly non-linear fields limits the amplitude of stable electron motion or dynamic aperture of the ring. The edge focusing introduces the additional quadrupole-like effects in the ring. It causes the change in betatron tunes or phases and the modulation of betatron functions called as -asymmetries or beta-beating. These perturbations break the very high symmetry for chromaticity correcting sextupoles distributed over the ring. The symmetry break can excite additional structural resonances and hence can further limit the dynamic aperture of the ring. Also, the strong -asymmetries can reduce the life time of the beam and can affect the other users of the ring due to the change in beam sizes. The non-linear fields of these devices produce the amplitude-dependent tune-shifts and can excite new resonances. The amplitude-dependent tune-shifts can drive machine operation near other resonance. It can reduce the dynamic aperture significantly and hence the life time of the beam. Therefore, it is very important to obtain the analytic descriptions as well as to understand theoretically the beam dynamics of a storage ring in presence of these devices.

2.1 Description of Magnetic field :

The magnetic field components of an ideal elliptic insertion device are given as;

$$B_x = B_0 (k_x/k_y)\, sh(k_x x) sh(k_y y)\, cos(kz) + B_o\, ch(k_x x)\, ch(k_y y)\, sin(kz)$$

$$B_y = B_0\, ch(k_x x)\, ch(k_y y)\, cos(kz) + B_o (k_y/k_x)\, sh(k_x x)\, sh(k_y y) sin(kz) \qquad (2.1.1)$$

$$B_z = -B_0 (k/k_y)\, ch(k_x x)\, sh(k_y y)\, sin(kz) + B_o (k/k_x)\, sh(k_x x)\, ch(k_y y)\, cos(kz)$$

Where : x, y are transverse co-ordinates and z is the axis of device or longitudinal co-ordinate.
B_0 and B_0 are the horizontal and vertical magnetic fields on the axis of the device.
The magnetic field must satisfy the Maxwell's equations and therefore;

$$div\, \mathbf{B} = 0$$

This leads to the following relations;

$$k_x^2 + k_y^2 = k^2 \quad and \quad k_x^2 + k_y^2 = k^2$$

Where : k = 2 / and is the period length of an insertion device.
The different names are given depending upon the fields B_o, B_o of a device and are as follows;

a. $B_o = 0$: *horizontal planar insertion device*, in which on axis particle oscillates (wiggle) in the horizontal plane (x-s plane).
b. $B_o = 0$: *vertical planar insertion device*, in which the on axis particle oscillates in the vertical plane (y-s plane).
c. $B_o = B_o$: *helical insertion device*, in which the on axis particle oscillates in both planes with same amplitude(helical wiggling).
d. B_o B_o : *elliptic insertion device*, in which on axis particle oscillates in both planes with different amplitudes (elliptic helical wiggling).

2.2 Beam Dynamics :

The beam dynamics with insertion devices can be understood well using a Hamiltonian formalism. A Hamiltonian for an electron motion around a wiggling orbit in a planar insertion device was first derived by L. Smith[1]. Later on, it was derived for helical insertion device by L. Tosi[2]. In order to understand the beam dynamics with pure elliptic devices for SLS as it will employ many advance insertion devices, a Hamiltonian for pure elliptic case has been derived by B. Singh et al.[3] from first principles. The planar and helical are the limiting cases of it. A Hamiltonian for helical case can be obtained from it under condition $B_o = B\ _o$. Both of them contain the same polynoms in x,y except their coefficients which are different. Therefore, the nature of beam dynamics is same in two cases. A Hamiltonian for helical case in which the hyperbolic functions are expanded up to fourth power in x, and y, is written here due to simple algebra for understanding of beam dynamics.

$$H = (p_x^2+p_y^2)/2 + \{(k_x^2+k\ _x^2)x^2 + (k_y^2 + k\ _y^2)y^2 + (k_x^4/3+k\ _x^4/3)x^4 + (k_y^4/3 + k\ _y^4/3)y^4 +$$

$$k^2(k_x^2 + k\ _y^2)x^2y^2 \}/(4k^2\ ^2) - \cos(kz)\sin(kz)/(k^2\ ^2)\ \{(k\ _y^2 + k_x^2)xy + (k^2k\ _y^2/2 + k\ _y^4/6 +$$

$$k_x^2k_y^2/6)xy^3 + (k^2k_x^2/2 + k\ _x^2k\ _y^2/6 + k_x^4/6)x^3y \} - p_x \{ \sin(kz)/(\ k)[k_y^2y^2/2 + k_x^2x^2/2 +$$

$$k_y^4y^4/24 + k_x^4x^4/24 + k_x^2k_y^2x^2y^2/4] - \cos(kz)/(\ k)\ [k\ _y^2xy + k\ _y^4xy^3/6 + k\ _x^2k\ _y^2x^3y/6]\}$$

$$-\ p_y \{\cos(kz)/(\ k\ )[k\ _y^2y^2/2 + k\ _x^2x^2/2 + k\ _y^4y^4/24 + k\ _x^4x^4/24 + k\ _x^2k\ _y^2x^2y^2/4]\ -$$

$$\sin(kz)/(\ k)\ [k_x^2xy + k_x^2k_y^2xy^3/6 + k_x^4x^3y/6]\} \qquad (2.2.1)$$

Where : is the radius of curvature corresponding to B_0 and is given by;
= 3.3356 E_0(GeV)/B_0(T) : E_0 *: design energy of machine*

The equations of motion governing the betatron oscillations can be derived from above Hamiltonian using Hamilton's equations. But the features of beam dynamics can be understood from Hamiltonian itself. The H contains the non-oscillating and oscillating terms (cos(kz), sin(kz) and cos(kz)sin(kz)).

Non-oscillating Terms :

i. The linear terms introduce the quadrupole-like effects or focusing in x,y. The focusing strengths or quadrupole gradients (K_x, K_y) are given by;

$$K_x = \ (k_x^2 + k\ _x^2)/2k^2\ ^2\ ,\ K_y = \ (k_y^2 + k\ _y^2)/(2k^2\ ^2) \qquad (2.2.2)$$

Therefore, an ID act as a pseudo quadrupole which has different gradients in x, y than usual one.

ii. The non-linear terms introduce the systematic octupole-like non-linearities. Therefore, IDs produce amplitude-dependent tune-shifts and excite the following 4th order resonances;

$$4Q_x = M\ ,\ 4Q_y = M\ ,\ 2Q_x \pm 2Q_y = M \qquad (M : integer)$$

Oscillating Terms :

i. The oscillating terms contain the normal and skew sextupole, octupole-like non-linearities and are equal to zero if averaged over a period length of the device. Therefore, their average effects may be negligible.

ii. The amplitudes of oscillations of sextupole-like fields averaged over a half period of the device are very large. They are comparable to that of the chromaticity correcting sextupoles of the ring and therefore, their effects are not negligible. They can excite intrinsic odd order sextupole resonances.

iii. This amplitude of oscillations of octupole-like fields is small and therefore its average effects are small and can be ignored.

A Hamiltonian for traditional planar IDs (L. Smith) in which electrons entering on axis and wiggle in a horizontal plane, can be obtained by setting k_x, k_y and all cosine terms to zero in (2.1.2).

$$H = (p_x^2 + p_y^2)/2 + (K_x x^2 + K_y y^2)/2 + [k_x^4 x^4 + k_y^4 y^4 + 3k_x^2 k^2 x^2 y^2]/(12k^2 \ ^2)$$
$$- \sin(kz) [p_x (k_x^2 x^2 + k_y^2 y^2) - 2k_x^2 p_y xy]/(2k \ ) \qquad (2.2.3)$$

Where : K_x and K_y are the linear focusing functions and are given as;

$$K_x = k_x^2/ (2k^2 \ ^2) , \qquad K_y = k_y^2/ (2k^2 \ ^2)$$

The oscillating octupole-like fields terms have been omitted. In case the coupling k_x is small, the focusing occurs in vertical plane only. It excites 4th order resonance $4Q_y$ = integer.

Therefore, IDs produce linear and non-linear effects on an motion of an electron when installed in the storage ring. The linear effects produce linear tune-shifts and distort the -functions of the machine (-asymmetries or beta-beating). The change in linear tunes can bring machine operation in near resonance. While -distortions break the strong symmetry of chromaticity correcting sextupoles distributed over the machine. Hence excite the additional systematic resonances. These effects shrink further the already limited dynamic aperture of the ring. The non-linear effects produce the amplitude-dependent tune-shifts and excite its own additional resonances or bring operation near other resonances due to amplitude-dependent tune-shifts. These effects can be quantified by estimating linear tune-shifts, -distortions or -asymmetries, non-linear amplitude-dependent tune-shifts, distortions of phase spaces and dynamic aperture. The excitations of additional resonances can be identified from phase space plots or FFT of numerical trackings of a particle.

2.3 Linear Tune-Shifts and -asymmetries :

The expressions for linear tune-shifts and -asymmetries up to first order can be derived by considering quadrupole-like effects as gradient errors and are given as;

$$Q_{x,y} = 1/4 \quad {}_{x,y} K_{x,y}\, ds$$

$$= (1/(8\ \ k^2\ \ ^2)) \quad {}_{x,y} (k_{x,y}{}^2 + k_{x,y}{}^2)\, ds$$

$$(1/(8\ \ k^2\ \ ^2))\ (\ \ {}_{x,y})_{av}\, L\, K_{x,y} = (\ \ {}_{x,y})_{av}\, B_0^2\, L\, K_{x,y} / ((3.3356 E_0\ )^2 .8\ \ k^2)$$

Where : ($_{x,y}$)$_{av}$ is the average -functions of the machine at the location of ID averaged over its length L.
The relative -asymmetries at the location j of the machine are given by;

$$(\ \ {}_{x,y}/\ \ {}_{x,y})_J = 1/(2\sin(2\ \ Q_{x,y})) \quad {}_{x,y}(w) \cos[2Q_{x,y}\ (\ \ - \quad {}_{x,y}(w) - \ \ {}_{x,y}(j)\ \ )K_{x,y}\, dw$$

$$Q_{x,y}$$

Where : w is the location of device.
The linear tune-shifts and -asymmetries are proportional to average -functions and $(B_0/E_0)^2$. Therefore, these effects will be more prominent for high field devices and in low energy machines. In order to minimize the linear distortions, they are to be installed in the location of low -functions of the machine. The third generation storage rings are designed to provide dispersion free straight sections with low -functions for installations of IDs.

2.4 Amplitude-Dependent Tune-Shifts and Non-Linear Resonances :

The non-linear terms in the Hamiltonian are responsible for non-linear effects and are proportional to $k^2/\ \ ^2$ or $(B_0/(E\ \ ))^2$. Since the undulators have low ., the non-linear effects are more prominent in undulators than wigglers. Also, the high field undulators break the strong symmetry of sextupoles due to linear effects. The average H contain the octupole-like and prominent oscillating sextupole-like non-linear fields and produce the amplitude-dependent tune-shifts. The amplitude-dependent tune-shifts can be derived using first and second order perturbation theories for octupoles and sextupoles respectively. Both of them are proportional to square of amplitude. These are measure of severity of non-linear effects introduced in the machine. In practice, they can be calculated from the numerical trackings.
The octupoles excite the 4th order non-linear resonances predicted from the first order perturbation theory.

$$4Q_x = M\ ;\quad 4Q_y = M\ ;\quad 2Q_x + 2Q_y = M\ ;\quad 2Q_x - 2Q_y = M \qquad (M : integer)$$

In case of SLS, the excitations for 4th order resonances $4Q_x = 83$ and $4Q_y = 33$ have been visualised by EEW in double undulator mode (2xUE212) in absence of sextupoles. The phase space plots have been obtained from numerical trackings after shifting linear tunes near to the resonances (fig.1 and fig.2). These plots show the four islands or occurrence of 4th order resonances.

The sextupoles excite the odd order resonances. It will be discussed later in this report that the effects of the oscillating sextupole-like fields is cancelled out if matched end poles are taken into account.

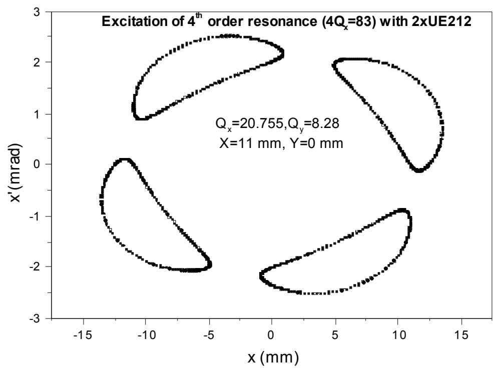


Fig.1 Horizontal phase space showing excitation of 4th order resonance $4Q_x = 83$ with 2xUE212 without sextupoles when linear $Q_x = 20.755$

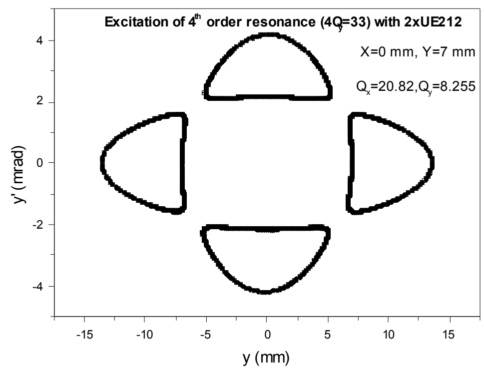


Fig.2 Vertical phase space showing excitation of 4th order resonance $4Q_y = 33$ with 2xUE212 when linear

3. *Simulation Models :*

The influences of IDs on beam dynamics of storage rings are studied using numerical simulations. The various simulation models have been developed at different laboratories. These tools are available in standard optics computer codes BETA and RACETRACK used for design of storage rings. It is important to know these models in order to obtain the required input parameters as well as for understanding of beam dynamics. The two models used for simulations studies of APPLE II/Sasaki-type and EEW or planar devices, are described here briefly.

3.1 *Simulation Model for APPLE-II or SASAKI-type Elliptic Undulator :*

It has been developed at BESSY-II by G. Wüestefeld et al.[4] for simulations studies of Sasaki-type or APPLE-II device. It is installed to provide the photon beams of any polarization from circular to linear in both horizontal and vertical planes. It is operated in double undulator mode, in which two halfs of one device is used to generate photon beams of different polarization separated by each other.
The undulator is consist of 4 long parallel beams of alternating rows of permanent magnets. A variable gap between the upper and the lower rows produces the different magnetic fields on the axis of device. The upper left and the lower right rows can be shifted with respect to other two rows. This phase shift results in magnetic field of arbitrary ellipticity. The scalar potential V of periodic part of the field is obtained by superposition of the contributions of four rows.

$$V = b_0 (V_1 + V_2 + V_3 + V_4)/8 , \quad \text{with}$$

$$V_1 = (e^{+k_y y}\, cx_- / k_y \; + \; e^{+ky} / k)\; sz_+$$

$$V_2 = (e^{+k_y y}\, cx_+ / k_y \; + \; e^{+ky} / k)\; sz_-$$

$$V_3 = (e^{-k_y y}\, cx_+ / k_y \; + \; e^{-ky} / k)\; sz_+$$

$$V_4 = (e^{-k_y y}\, cx_- / k_y \; + \; e^{-ky} / k)\; sz_-$$

and $cx_\pm = \cos(k_x (x \pm x_0))$, $sz_\pm = \sin(kz \pm \;\; /2)$.

The other parameters are defined as;
$_x = 2\;\; /k_x$, $_y = 2\;\; /k_y$, $= 2\;\; /k$, $k_y^2 - k_x^2 = k^2$
(is the longitudinal period length of an insertion device)

It is to be noted that k_x is different from previously defined ($k_x =$ ik_x (previous k_x)).

b_0 : field strength parameter
x_0 : horizontal gap separation
: shift parameter and is the measure of shift of the two rows with respect to other two.

The magnetic fields on the axis can be varied by changing the shift parameter from 0 to 2 . The fields on axis can be obtained from above scalar potential and given as;

$$B_y\,(x = 0, y = 0, z) = -b_0\,(1 + \cos(k_x x_0))\,\cos(\;\;/2)\,\sin(kz)$$

$$B_x\,(x = 0, y = 0, z) = -b_0\, k_x \sin(k_x x_0) \sin(\;\;/2)\,\cos(kz)/(2k_y)$$

The different fields are produced on the axis of device depending upon the value of . It acts as horizontal and vertical planar device for = 0 and respectively.

= 0 , $B_x = 0$ ----- plane device --- horizontal -wiggle

= , By = 0 ---- plane device --- vertical -wiggle

In case of UE56 device of SLS, the maximum fields on the axis (x_0 = 0.02m, = 0.056m and b_0 =1.3 T) are as follows ;

$B_{y0} = 0.76$ T , $B_{x0} = 0.34$ T

This tracking routine needs b_0(T), $_x$ (m), s (s = /2), E_0(GeV) and its length(m) as input parameters. The end poles are defined separately on either side of the device and treated as separate IDs. They are integrated by step generating function. This model is incorporated for numerical simulations in optics code **BETA** (BESSY version). Since the end poles are treated separately, it can be used to understand the importance of end poles on a motion of an electron in a storage ring.

3.2 Simulation Model for Elliptic Electromagnetic and Planar Undulators :

The various simulation models are available in computer code RACETRACK for elliptic device based on definitions of various field distributions. The realistic field distribution can not be represented accurately by hyperbolic functions because of unusual geometry of real poles. Therefore, another model was developed by L. Tosi et al.[5] in which the transverse field distributions are expressed in form of polynomials of sixth order. This model was used for simulations studies of elliptic electromagnetic wiggler (EEW) at ELETTRA. In this model, the suitable field distribution is obtained using polynomial expansions for the off axis variation in the following form:

$$\phi_h = \sum B_h f(x,y)\sin(kz)$$

$$\phi_v = \sum B_v g(x,y)\cos(kz)$$

with

$$f(x,y) = x+a_{2h}x^3+b_{2h}y^2x+a_{3h}x^5+b_{3h}y^2x^3+\mathbf{c}_{3h}y^4x+a_{4h}x^7+b_{4h}y^2x^5+c_{4h}y^4x^3+d_{4h}y^6x$$

$$g(x,y) = y+a_{2v}y^3+b_{2v}x^2y+a_{3v}y^5+b_{3v}x^2y^3+\mathbf{c}_{3v}x^4y+a_{4v}y^7+b_{4v}x^2y^5+c_{4v}x^4y^3+d_{4v}x^6y$$

Where : $_h$, $_v$ are the scalar potentials corresponding to the horizontal and vertical poles respectively. An elliptic device can be considered as a device made of two horizontal and vertical planar devices and corresponding transverse field components can be obtained in followings way;

$$B_y(x,0,s) = - \quad _v / \quad y = B_{vn}(1 + b_{2v}x^2 + c_{3v}x^4 + d_{4v}x^6)\cos(kz)$$

$$B_x(0,y,s) = - \quad _h / \quad x = B_{hn}(1 + b_{2h}y^2 + c_{3h}y^4 + d_{4h}y^6)\sin(kz)$$

The two sets of coefficients (b_{2v},c_{3v},d_{4v}), (b_{2h},c_{3h},d_{4h}) are obtained by fitting the expressions to Fourier-analyzed computed field distributions. and other coefficients are calculated from Maxwell's equation.

$$\text{div}\,\mathbf{B} = 0$$

In case of planar device, only one set of coefficients b_2,c_3,d_4 is required. The field distribution in case of horizontal planar device (vertical poles) is given as;

$$B_x = B_o\,(2b_2\,xy + 2b_3\,xy^3 + 4c_3\,x^3y + 6d_4x^5y + 4c_4\,x^3y^3 + 2b_4\,xy^5)\cos(kz)$$

$$B_y = B_o(1 + 3a_2\,y^2 + b_2\,x^2 + 5a_3\,y^4 + 3b_3\,x^2y^2 + c_3\,x^4 + 7a_4\,y^6 + 3b_4\,x^4y^2 + 3c_4\,x^4y^2 + d_4\,x^6)\cos(kz)$$

$$B_z = -kB_0(y + a_2\,y^3 + b_2\,x^2y + a_3y^5 + b_3x^2y^3 + c_3x^4y + a_4y^7 + b_4x^2y^5 + c_4x^4y^3 + d_4x^6y)\sin(kz)$$

Here suffix v omitted.

In the median plane (y = 0) and at cos(kz) = 1, only vertical field B_y exists.

$$B_y = B_0\,(1 + b_2\,x^2 + c_3\,x^4 + d_4\,x^6)$$

The coefficients b_2,c_3,d_4 are obtained from the field data calculated from simulation code or from measured field data by polynomial fit. In case of elliptic electromagnetic device, the two sets of these coefficients are obtained for field distribution corresponding to horizontal and vertical planar devices. This model is implemented for numerical simulations in computer code RACETRACK. It requires b_2,c_3,d_4 and B_0, B_0 as input parameters. This model has been used to simulate the effects of 2xUE212 and all planar devices for SLS[#] .

4. *End Poles :*

In order to obtain the stable electron motion in a storage ring in presence of ID, it is essential to have matched end poles on either sides of the device. The end poles match the closed orbit from the outside of the orbit to a wiggling periodic orbit inside a device and back to outside. In ideal condition, it is achieved without any residual kick or displacement of the beam. This is achieved when two field integrals of both transverse field components B_y and B_x are zero or:

$$I_1 = \int_{-L}^{L} B_{x,y}\,dz = 0 \qquad and \qquad I_2 = \int_{-L}^{L}\int_{-L}^{z} B_{x,y}\,dz'\,dz = 0$$

[#] Fields have 10 and 14 poles -like multipoles which do not appear in Hamiltonian. If hyperbolic functions are expanded up to 7[th] power in x, y, these terms will appear.

It can be achieved by pure dipole fields without any higher order field components. But in practice, is achieved by real poles. It is described in a paper by G. Wüestefeld et al.[4] that the effects of oscillating sextupole-like fields can be cancelled out by properly choosing the end pole configurations in addition to matching of closed orbit. It requires that the betatron phase advances should be small and -functions are constant over the ID. The magnetic fields of an elliptic device contains both cos(kz) and sin(kz) terms. The cos(kz)-like terms is naturally matched for both closed orbit and as well as for sextupoles however sin(kz) -like term requires a special end poles at least.

4.1 Treatment of End Poles in Simulation Codes :

The proper treatment of end poles in numerical simulation models is very important for realistic estimations of their effects. In initial numerical simulations models of computer code RACETRACK, it is done by shifting the co-ordinates of the particle from the reference orbit of the machine to a periodic wiggling orbit of an ID at entry and back at the exit. It is done by shifting the co-ordinates of the particle at entry and exit by an amount:

$$x = 1/(k^2 \quad ^2), \quad x = 0, \qquad y = 0, \qquad y = -1/(k \quad)$$

where : & are the radius of curvatures corresponding to fields B_0 and B_0.
This treatment is equivalent to consider the end poles as dipoles. After findings of G. Wüestefeld, the numerical tracking routines were modified. The two codes (RACETRACK and BETA) have adopted different approaches. RACETRACK has adopted the approach of the continuity of conjugate momentum by adjusting the slope properly. While BETA code has included an end pole configuration on either side of device and consist of two poles of reduced field amplitudes. These end poles are integrated by a step generating function over a half period length. The matched end poles configuration which is included in computer code BETA , are defined as;

End pole configuration (half poles of different amplitudes)

1st	2nd	
1/4	-3/4	--- entry
3/4	-1/4	---- exit

These poles have been treated as separate IDs.
In case of planar ID, the fields contain only cos(kz) terms and therefore, it is naturally matched for both the closed orbit as well as sextupoles of the ID. So, it does not require to have special end poles.
BETA code can be used to understand the influences of end poles on beam dynamics. In order to understand it, the numerical tracking simulations have been done with different initial particle co-ordinates in presence and absence of chromaticity correcting sextupoles with and without end poles for the low emittance SLS optics mode (Q_x = 20.82, Q_y = 8.28) with APPLE-II device in double undulator mode (2xUE56). The following conclusions are drawn from tracking results presented in fig.3, fig.4 and fig.5.

a. There will not be a stable motion in presence of chromaticity correcting sextupoles if ID has no end poles in both planes (x, y) due to severe coupling as is seen in fig.3 and fig4. The horizontal motion with (x, x) = (15 mm, 0 mrad), it fills whole phase space and get lost while vertical motion with (y, y_i) = (3mm, 0 mrad) blows up fast.

b. The particle with co-ordinates (x, x , y, y) = (16 mm , 0 mrad, 16 mm, 0 mrad) is simulated with and without end poles in absence of sextupoles for 1000 turns and presented in fig.5. The solid lines and dots are for with and without end poles respectively. In case of ID with end poles, it is perfect circle while good degree of coupling is noticed in case without end poles. It occurs due to the effects of oscillating sextupole-like fields of the ID. Therefore, the non-linear effects of an ID are reduced significantly if end poles are included properly in numerical simulation model.

c. It is to be noted that the chromaticity correcting sextupoles of the machine make the electron motion unstable in absence of end poles.

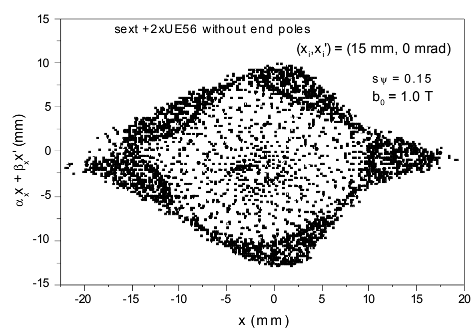


Fig.3. Horizontal normalised phase space for tracking of a particle with initial co-ordinates (x_i,x_i) = (15 mm, 0 mrad) with sextupoles and 2xUE56 without end poles.

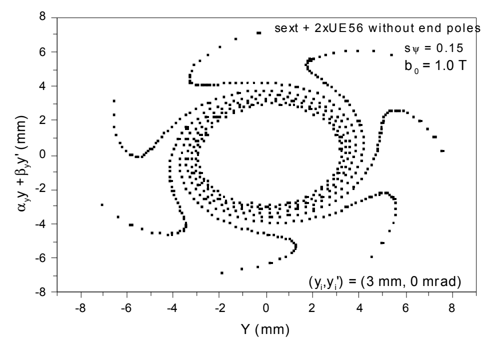


Fig.4 Vertical normalised phase space for tracking of a particle with initial co-ordinates (y_i,y_i) = (3 mm, 0 mrad) with sextupoles and 2xUE56 without end poles

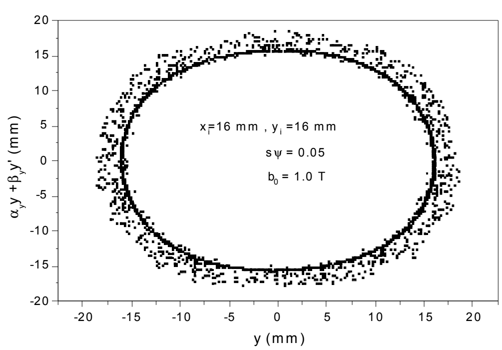


Fig.5 Vertical normalised phase space for tracking of a particle with initial co-ordinates (x_i, x_i,y_i, y_i) = (16 mm, 0 mrad, 16 mm, 0 mrad) in absence of sextupoles with (solid lines) and without (dots) end poles.

5. Field Errors :

The above descriptions of IDs are based on ideal field models. In practice however the magnetic field errors arise due to the variations in the shape and dimensions of the magnets, poles and variations in gaps and magnetisation strength and angle from block to block and inhomogeneity. These errors will further perturb the motion of an electron beam. Ideally, the first and second integrals must be zero but in reality these integrals are not zero. The non-zero values of these integrals along the optical axis will give rise to non zero closed orbit and variations with transverse co-ordinates These integrals can be expressed in terms of multipole components which produce the change in linear tune values and betatron functions (normal quadrupole errors), change in coupling (skew-quadrupole errors)and as well as non-linear effects (sextupole and octupole errors etc). The effects of various field errors have been observed in various storage rings[6]. The effects of field errors on beam dynamics of SLS have been studied separately to set the tolerances for field errors of IDs[7].

6. Brief of Effects of Insertion Devices on Beam dynamics :

The perturbations produced by IDs can be summarised as below;
The IDs produce the linear and non-linear perturbations on the beam dynamics of storage ring.

Linear Effects:

i. Proportional to $(B_0/E)^2$ and -functions of the machine.
ii. Produced by the wigglers due to their high field or high field undulators.
iii. They are severe in low energy storage rings and if installed at high locations of the ring.
iv. Produce the linear tune-shifts and modulation of -functions (-asymmetries) around the rings.
v. -asymmetries cause the change in beam sizes and if large, can affect the beam life time and users of synchrotron radiation.
vi. Reduce the dynamic aperture of the ring due to the break of high symmetry of sextupoles distributed over the ring and can excite additional sextupole resonances or can bring machine operation near to the other sensitive resonances of the ring.

It may be essential to compensate linear tune-shifts and -asymmetries for smooth operation of machine globally, locally or both.

Non-Linear Effects :

i. Proportional to $(B_0/E\ )^2$ and -functions of the machine.
ii. Produced mainly by undulators due to their low period length ( ).
iii. They are severe in low energy rings for high field undulators with low period length and located in high regions.
iv. Introduces its own non-linearities that can excite additional resonances and produce the amplitude-dependent tune-shifts.. Thus can limit the dynamic aperture further.
v. Excite the 4th order regular resonances $4Q_x = M$, $4Q_y = M$ and $2Q_x \pm 2Q_y = M$ (M : integer)

vi. The regular odd order sextupole resonances ($3Q_x = M$; $3Q_y = M$; $Q_x + 2Q_y = M$; $Q_x - 2Q_y = M$; $Q_y + 2Q_x = M$; $Q_y - 2Q_x = M$.) can also be excited by strong oscillating sextupole-like fields if conditions of matching failed.

7. *Numerical Simulations Studies for SLS :*

It is planned to operate two elliptic and three planar device in SLS. An Apple-II or Sasaki-type (UE56) and a elliptic electromagnetic wiggler (UE212) will operate in double undulator mode and located in a medium (11 M) and a long (9L) straight sections respectively. The Planar devices a wiggler (W61) and two in-vacuum undulators (U24, U17) will be installed in short straight sections 4S and 6S respectively. The U24 will be replaced by U17 later. The parameters of these devices[8] are listed in table.1.

Table.1

ID	Total length (m)	No. of periods (Np)	Period Length (mm)	B_y (T)	B_x (T)	Gap (mm)	Type
UE212	10	2 x 21	212	0,45	± 0,1	19	EMAG
UE56	6	2 x 33	56	0.83	± 0.6	16	Apple-II/Sasaki-type
U24	2	80	24	1.0		6	Hybrid
U17	2	110	17	1		4	Hybrid
W61	2	33	61	2		7.5	Hybrid

Non-Linear Beam Dynamics with sextupole :

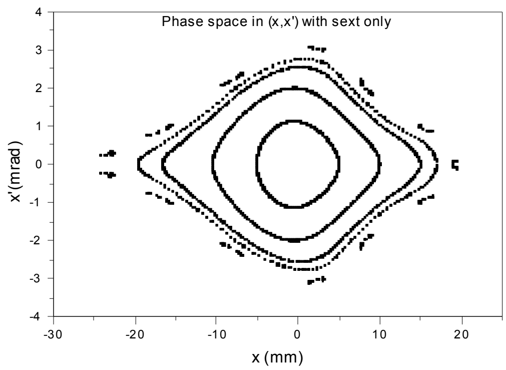
Some features of non-linear beam dynamics of SLS for low emittance optics mode ($Q_x = 20.82$ and $Q_y = 8.28$) with chromaticity correcting sextupoles as well as some other noticed during numerical investigations are mentioned below.

i. The horizontal phase space plot (fig.6) shows no appearance of any resonance either stable or unstable up to border of stability.

i. The vertical phase space plot (fig.7) shows the occurrence of the coupling only at the border of stability (y = 6 mm and 7 mm).

ii. The vertical dynamic aperture is sensitive to a small increase in a vertical tune Q_y (fig.8). It falls from 7 mm to 4 mm when it is moved from $Q_y = 8.28$ to 8.29. Therefore, dynamic aperture is calculated for IDs with compensation of linear tunes to avoid it due to their linear effects.

iii. It is safe to calculate the dynamic aperture from bounded to unbounded region as there is unstable region between stable regions (fig.9). This has been noticed during numerical investigations for 2xUE212 undulators of SLS. Similar features have been noticed in SUPER-ACO[6]. The islands lie in this region are unstable. The particles in this region are lost in different number of turns and therefore horizontal DA changes with number of turns (fig.10).



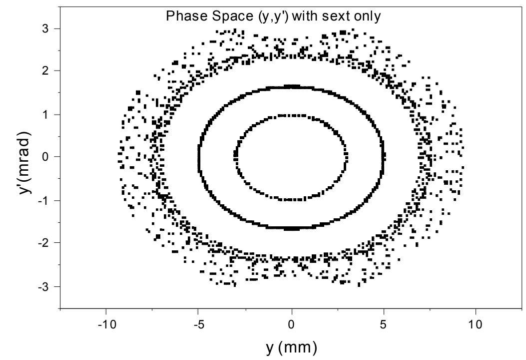


Fig.6 Horizontal phase space upto border of stability

Fig.7 Vertical phase space upto border of stability

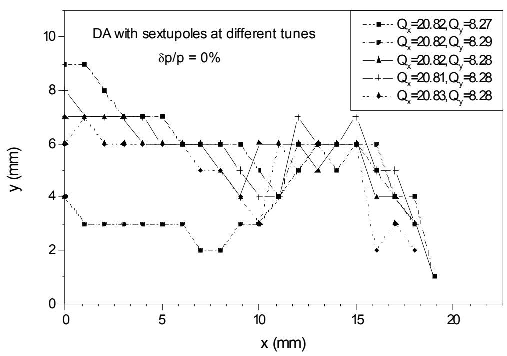


Fig.8 Dynamic aperture (DA) for sextupoles only with different tunes

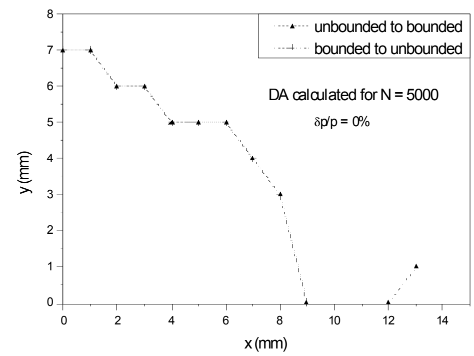


Fig.9 Dynamic aperture calculated from bounded to unbounded and unbounded to bounded regions

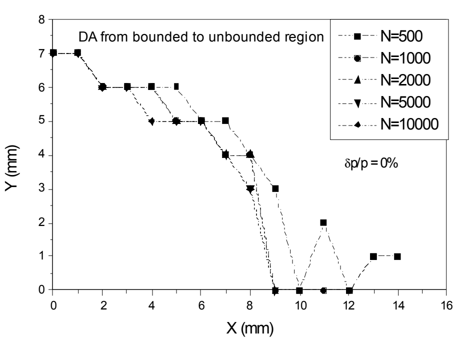


Fig.10 Dynamic aperture for different number of turns (N)

7.1 APPLE II/Sasaki-type undulators (2xUE56) :

This device consists of twin-undulators (double undualtor mode). Each of them are used to produce different. polarized photon beams. The electron beam is displaced parallel by 1 mm in two undulators using small dipole magnets to separate radiation from them. They can be tuned to obtain any degree of polarization from linear to circular in both horizontal and vertical. Its influences on the beam dynamics has been investigated using simulation model developed at BESSY-II and implemented in BETA Code. The general parameters of this device are listed in the table1. Other parameters# required for numerical simulations are;

b_0 (remanant magnetic field) = 1.3 T ,   = 2 /k = 56 mm , $_x$ = 2 /k_x = 89.6 mm,
$_y$ = 2 /k_y = 47.5 mm, x_0 = 20 mm, Np (number of periods) = 33 , s = 0 to 1

Linear Distortions :

The linear tune-shifts and -asymmetries are produced due to fields on the optical axis and are proportional to square of it. A variable gap between the upper and the lower rows produces the different fields on axis. The maximum on axis fields (both horizontal B_x = 0.34 T (s = 0.0) and vertical B_y = 0.76 T (s = 0.5)) are produced when these undulators operate as planar device. Therefore, the maximum linear distortions are produced when these undulators are operated in planar modes.

The linear tune-shifts are estimated from tracking as there is no provision in code to treat this device as a linear element. These are estimated from tracking of a particle with a small initial amplitude and -asymmetries from the fact that these are directly proportional to linear tune-shifts.

Linear tune-shifts :

Q_x = 0.0027 for s = 0.5 ----- vertical planar device

Q_y = 0.0045 for s = 0.0 ----- horizontal planar device

-asymmetries = ($_{x,y}$/ $_{x,y}$)$_{max}$ < 4 %

The maximum -asymmetries occur in vertical plane and are less than 4% when both undulators operate as horizontal planar device. The symmetry loss is insignificant. Also, This mode produces maximum vertical linear tune-shift. It has been already mentioned that the dynamic aperture (for sextupoles) is sensitive to a small increase in vertical tune. The dynamic aperture gets affected most in this case if linear increase in Q_y is not corrected.

The linear tune-shifts are calculated first and then are corrected using quadrupole families QLF and QLG. Afterwards, the dynamics aperture is calculated (fig.17) which shows that it recovers back.

Non-Linear Effects :

The non-linear effects are estimated by the amplitude-dependent tune-shifts and the deformations of circular trajectories in the normalized phase space. They are also proportional to square of on axis fields. Therefore, they are maximum when both undulators operate in planar (horizontal or vertical) modes. In order to quantify them, the amplitude-dependent tune-shifts are calculated from the trackings of a particle for planar modes (fig.11 and fig.12) due to device only (no sextupoles). These are insignificant and are less than 10^{-3} in the region of interest# for SLS.

The parameters required for numerical simulations of different devices have been provided by ID group (G.Ingold & T.Schimdt) at SLS.

Also, the phase space plots have been obtained from trackings for a case s = 0.15 for both undulators with device only (fig.13 and fig.14). They do not show any noticeable deformations of circular trajectories in this region. No noticeable difference is observed between phase spaces of sextupoles only (fig.6, fig.7) and sextupoles with device (fig.15, fig.16).
Therefore, it can be concluded that device produces a very weak non-linear effects.

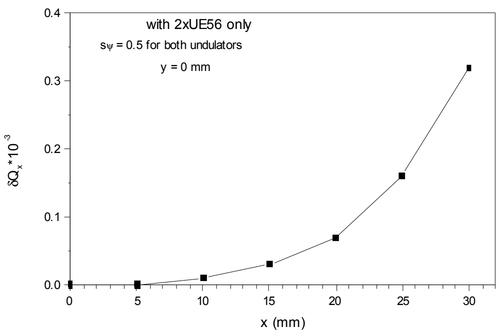


Fig.11 Horizontal amplitude-dependent tune-shift with 2xUE56 without sextupoles when both undulators operate as vertical planar device.

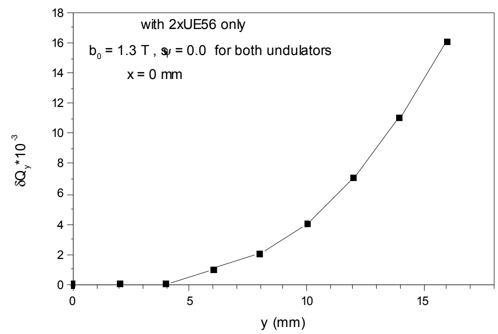


Fig.12 Vertical amplitude-dependent tune-shift with 2xUE56 without sextupoles when both undulators operate as horizontal planar device.

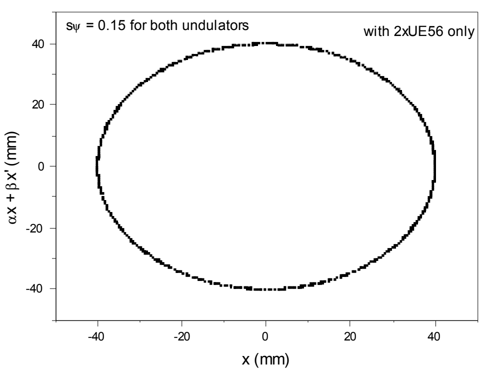


Fig.13 Normalized horizontal phase space for tracking of a particle (x,x) = (40 mm, 0 mrad) with 2xUE56 without sextupoles for shift parameter s = 0.15 for both undulators.

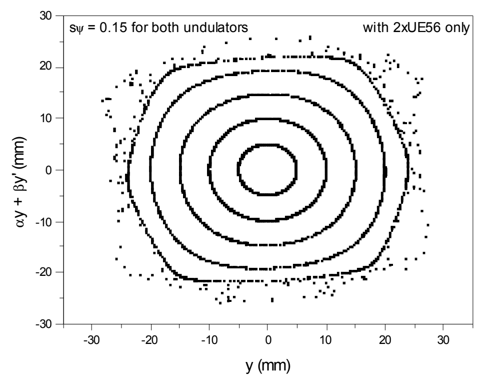
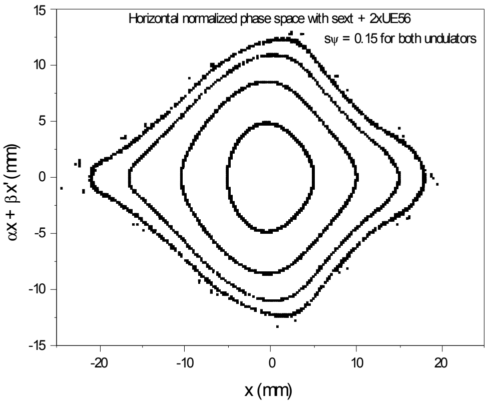


Fig.14 Normalized vertical phase space of tracking of a particle upto border of stability with 2xUE56 without sextupoles for shift parameter s = 0.15 for both undulators.

The *region of interest* is defined as dynamic aperture determined by the machine sextupoles only (x=19 mm, y=7 mm for SLS).



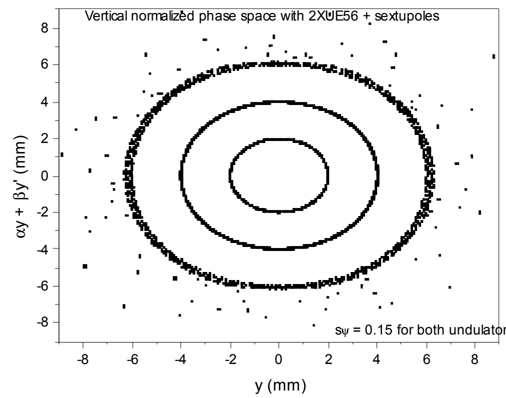


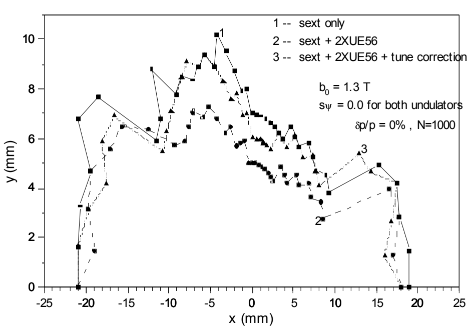
Fig.15 Horizontal normalized phase space upto the border of stability for 2xUE56 + sextupoles

Fig.16 Vertical normalized phase space upto border of stability for 2xUE56 + sextupoles

Dynamic Aperture:

To investigate the effects of this device on the dynamic aperture, one need to calculate it for entire range of shift parameter (0 to 2 or s = 0 to 1) with at least compensation of increase in linear vertical tune Q_y . It is already discussed that vertical dynamic aperture recovers after correction of vertical linear tune. The dynamic apertures have been calculated for p/p = 0% with various s (same for both undulators) and different values of s for two undulators without compensations of linear tunes (fig.18 and fig.19) . It can be seen from these figures that there is no significant change in horizontal while some reduction in vertical is noticed. It is due to increase in linear Q_y which recovers after correction.

Therefore, the effects of this device on the dynamic aperture of the ring can be ignored for entire range of shift parameter provided vertical linear tune-shift (linear effects) is compensated globally.



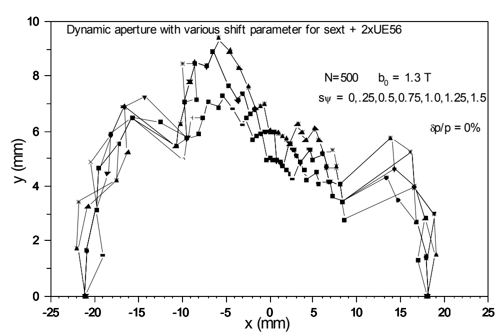


Fig. 17 Dynamic aperture for N = 1000 turns with sextupoles , sextupoles+2xUE56 , sextupoles + 2XUE56 with compensation of linear tunes

Fig18 Dynamic aperture for different s but same for both undulators with compensation of linear-tunes

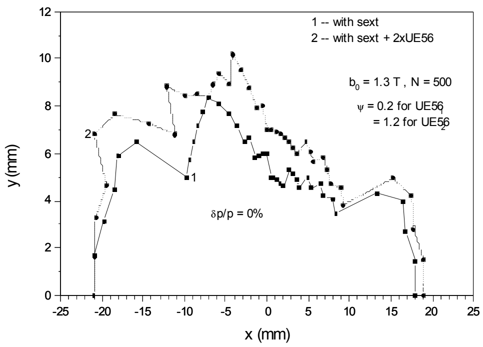


Fig.19 Dynamic aperture with different s for two undulators without compensation of linear tunes

Effects of Beam Displacement on Dynamic Aperture:

The electron beam is displaced parallel in horizontal plane inside two undulators using short dipole magnets (chicane magnets). To study the effect of beam displacement on the dynamic aperture, the parallel orbit bump is produced using kicker magnets located at the middle of these dipoles. No effect is observed on dynamic aperture even when orbit bump is 4 mm (fig.20). It is expected result. If beam is displace by 4 mm from orbit or axis of device inside undulators, it means beam will see higher non linear fields. But non-linear effects of device are very weak in horizontal plane for large amplitudes. Therefore, it does not affect the stability of particle and hence dynamic aperture.

Effects of Multipoles of Chicane Magnets on Dynamic Aperture:

Two types of dipole magnets are used for chicane and their multipole components are different[#] . The magnets to be installed at entry and exit (side) are of one type and at the middle are other type. The skew multipoles are insignificant and therefore, they have been ignored. The integrated strength of a normal multipole is defined as;

$$b_n l = (d^{n-1}B_y/dx^{n-1}).l\,/((n-1)!B\ \ )$$

Where 2n is the number of poles and l is the length. The multipoles are given in table.2.

Table.2

	Middle magnets	*side magnets*
2n	$b_n.l$	$b_n.l$
2	$4.67\ 10^{-3}$	$-5.93\ 10^{-3}$
4	$2.10\ 10^{-10}$	$-\ 4.60\ 10^{-4}$
6	$6.45\ 10^{-2}$	- 0.3085
8	$4.57\ 10^{-6}$	$-\ 7.40\ 10^{-4}$

[#] Data for multipoles of chicane magnets have been provided by T. Schimidt of ID group at SLS.

10	- 7.87	18.80
12	$3.96\ 10^{-3}$	$- 6.48\ 10^{-5}$
14	$5.66\ 10^{3}$	$3.28\ 10^{3}$
16	$6.68\ 10^{-1}$	$- 2.94\ 10^{-2}$
18	$-17.67\ 10^{5}$	$- 5.76\ 10^{5}$

The dynamic aperture is calculated in presence of these multipoles (fig.21) with orbit bump of 1 mm. The multipoles have no effect on dynamic aperture. It is noted during simulation studies that sextupole component of side magnet is significant and its effects are cancelled out due to opposite polarities. In case polarities are same, the horizontal dynamic aperture falls by 40% due to excitation of 5th order resonance $5Q_x = 104$.

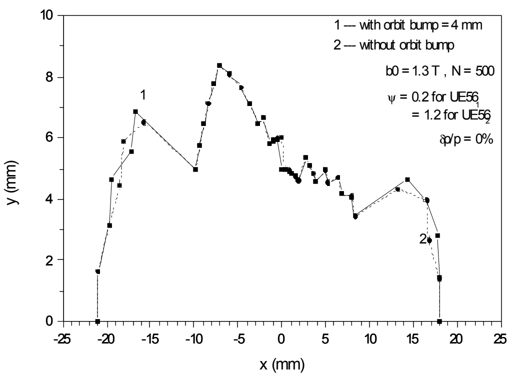


Fig.20 Dynamic aperture with and without orbit bump

(orbit bump = 4 mm)

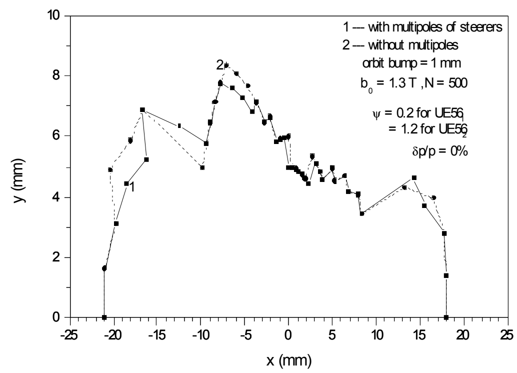
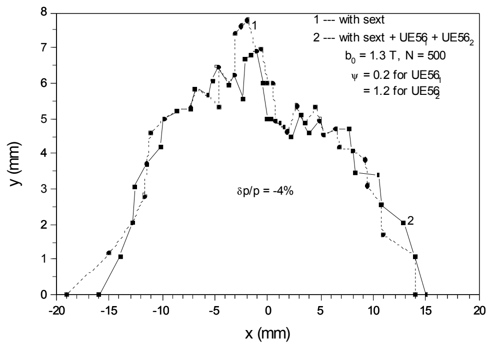


Fig.21 Dynamic aperture with and without multipoles

of dipole magnets of chicane

Dynamic Aperture for Off Momentum Particles :

The effect of this device on off- momentum particles is negligible even when linear tune-shifts are not compensated. The dynamic apertures for p/p = -4 % and +4% are shown in fig.22 and fig.23 respectively.



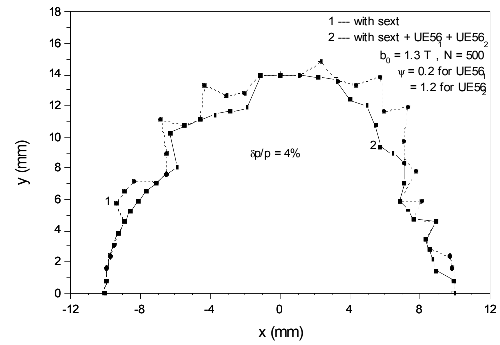


Fig. 22 Dynamic aperture for p/p = - 4% with sextupoles, with sextupoles+2xUE56 without linear tunes compensation linear

Fig. 23 Dynamic aperture for p/p =4% sextupoles,sextupoles+2xUE56 without tunes compensation

It is concluded that 2xUE56 produces weak linear and insignificant non linear effects. The maximum -asymmetries are less than 4% and maximum linear tune shift is produced in vertical plane and is less than 0.005. Some reduction in vertical dynamic aperture is observed and it is due to increase in a vertical linear tune due to its linear effects. It recovers back after increase in linear Q_y is compensated globally using quadrupole families QLF and QLG. No effect of beam displacement and multipoles of chicane magnets is noticed on dynamic aperture. Therefore, this device will not have any significant effect on the dynamic aperture for entire range of shift parameter( ).

7.2 Elliptic Electromagnetic Undulators (2xUE212) :

This device consists of twin electromagnetic elliptic undulators (2xUE212) with 21 periods of 212 mm length each. The twin-undulators can be operated either in the opposite or same helicity mode. The electron beam is displaced parallel by 1 mm same as in case of APPLE-II device with identical chicane magnets. The simulation model based on polynomial field descriptions of RACETRACK has been used to study its effects.

The parameters of this device required for simulations are as follows;

$B_0 = 0.5$ T , $b_2 = - 4.5\ 10^2$ [m^{-2}] , $c_3 = 3.38\ 10^4$ [m^{-4}] , $d_4 = 1.01\ 10^6$ [m^{-6}]
------- (vertical poles)
$B_0 = 0.08$ T , $b_2 = 2.46\ 10^2$ [m^{-2}] , $c_3 = 1.01\ 10^4$ [m^{-4}] , $d_4 = 1.66\ 10^6$ [m^{-6}]
------- (horizontal poles)
Np = 21 , = 212 mm
Linear distortions:
The linear tune-shifts and -asymmetries are estimated by treating it as a linear device.
Linear tune-shifts :
Q_x = -0.0091 , Q_y = 0.0166
-asymmetries :

The -asymmetries are calculated around the ring with and without corrections of the linear tune-shifts (fig.24 and fig.25). The linear tune-shifts are compensated globally using quadrupole families QLF and QLG.

-asymmetries	<	4%	without compensation of linear tune-shifts
	<	6%	with compensation of linear tune-shifts (except at middle of device)

The following are conclusions of linear perturbations;

i. -asymmetries are nominal (small).

ii. -asymmetries are more in case linear tune-shifts are compensated.

iii. The linear tune-shifts are small but not insignificant.

iv. The vertical linear tune Q_y is increased. It reduces the vertical dynamic aperture significantly but recovers after its compensation (already discussed) .

Non-linear effects :

The amplitude-dependent tune-shifts have been calculated from trackings (fig.34) for device only (no sextupoles). These tune-shifts are less than 0.001 in both planes in a region of interest for SLS. Therefore, non-linear effects are very weak and should not make any significant effect on beam dynamics.
The amplitude-dependent tunes calculated in presence of device and sextupoles are shown in fig.26 and fig.27.

Dynamic Aperture :

The dynamic apertures (p/p = 0%) have been calculated for both linear and non-linear devices after compensation of linear tune-shifts globally using quadrupole families QLF and QLG (fig.28). There is no significant reduction in dynamic aperture in vertical while it has reduced significantly in horizontal (19 mm to 10 mm for non-linear and 10 mm to 9 mm for linear). There is almost no difference between linear and non-linear cases. Therefore, this reduction in horizontal occurs due to loss of symmetry of the ring or -asymmetries.
In order to investigate the reasons, the normalized horizontal phase space plots have been obtained from trackings (fig.30 and fig.31) for linear and non-linear devices. The five islands appear in phase space plots at border of stability. This implies that the 5th order resonance $5Q_x = 104$ is excited. This resonance is unstable because particles on it are lost after certain number of turns. The horizontal amplitude-dependent tunes have been calculated from trackings for both linear and non-linear cases (fig.32). The horizontal fractional tune Q_x reduces from 0.82 to 0.80 due to amplitude dependence in both cases. Therefore, it reduces due to sextupoles and not because of non-linear fields of device. This change in horizontal amplitude-dependent tune occurs due to change in the -functions at the locations of sextupoles and betatron phases between them due to -asymmetries produced by a device. Therefore, this resonance is excited by a chromaticity correcting sextupoles. The 5th order resonances can be excited by the sextupoles which is predicted by third order perturbation theory[9].

Further, these tracking studies show that the five unstable islands lie between stable trajectories in phase space. The unstable region lies between stable regions. This type of behaviour with IDs was noticed in SUPERACO[6] also.

Effects of Opposite Helicity mode on Dynamic Aperture:

The twin undulators will be operated in opposite helicity mode also. This means the horizontal magnetic fields in two undulators will be in opposite directions. To investigate the effects of opposite fields, the dynamic apertures have been calculated with opposite horizontal fields ($B_x = \pm$). Theoretically, there should not be any change in nature of dynamics in case two undulators have opposite fields because all linear as well as non-linear effects are proportional to square of magnetic fields. The linear tune-shifts calculated are the same. Also, There is no difference in dynamic apertures calculated for two cases (fig.29).

Effect of Beam Displacement and Multipoles of Chicane Magnets :

This device produces a very weak non-linear effects similar to 2xUE56. Therefore, the displacement of beam in two undulators does not have any effect on dynamic aperture. The multipoles are the same as in case of 2xUE56. Also, they have no effect on dynamic aperture (fig.33).

It is concluded that though -asymmetries are nominal (small and lie within numerical and measurements accuracies) but produces the strong horizontal amplitude-dependent tune-shift. Thus reduces the horizontal dynamic aperture significantly. Therefore, the following observations are made;

i. *Tunes compensation should be done in a better way.*
ii. *Machine operating point $Q_y = 8.28$ is probably not good.*

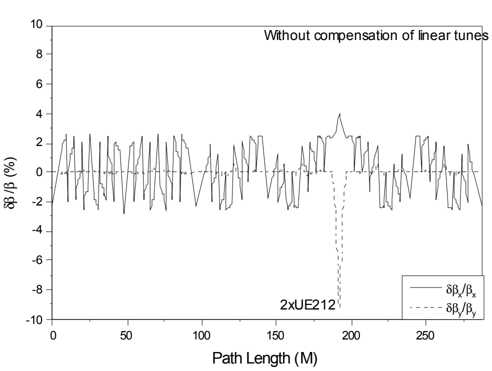


Fig. 24 -asymmetries with 2XUE212 without linear tunes compensation

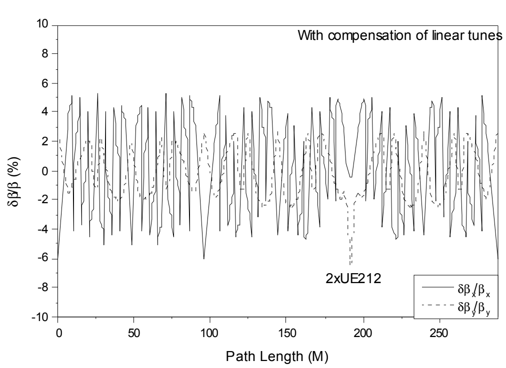


Fig.25 -asymmetries with 2XUE212 with linear tunes compensation

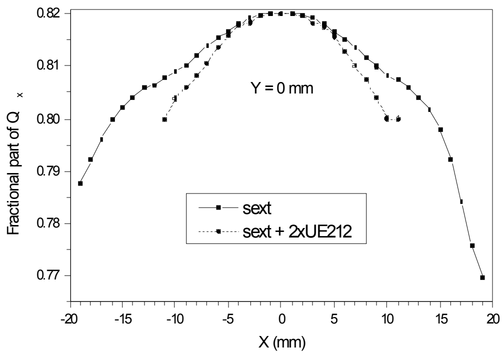


Fig.26 Fractional Part of amplitude-dependent horizontal tune for sextupoles and sextupoles + 2xUE212

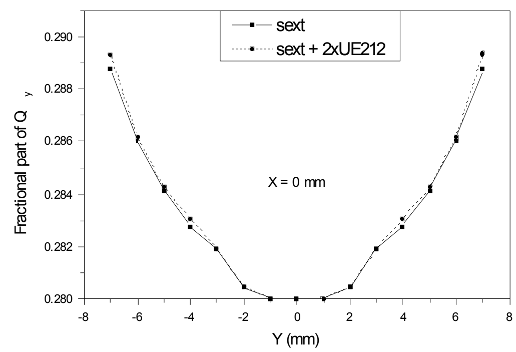


Fig.27 Fractional Part of amplitude-dependent vertical tune for sextupoles and sextupoles + 2xUE212

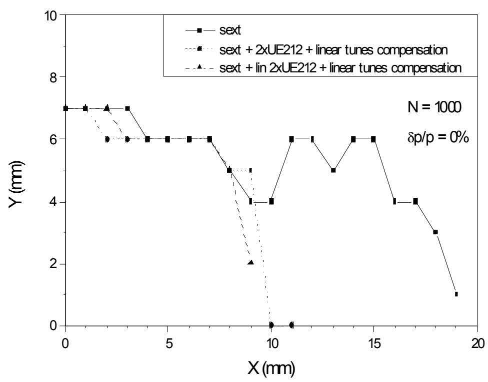


Fig.28 Dynamic aperture for sextupoles , sextupoles + 2xUE212, sextupoles + linear 2xUE212 when linear tunes are compensated (p/p = 0%)

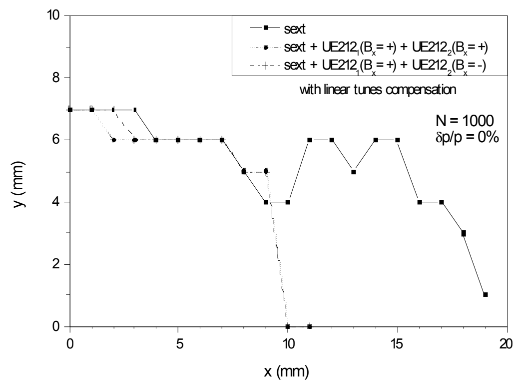


Fig.29 Dynamic aperture for sextupoles, sextupoles+2xUE212 with same and opposite B_x field component for two undulators (p/p =0%)

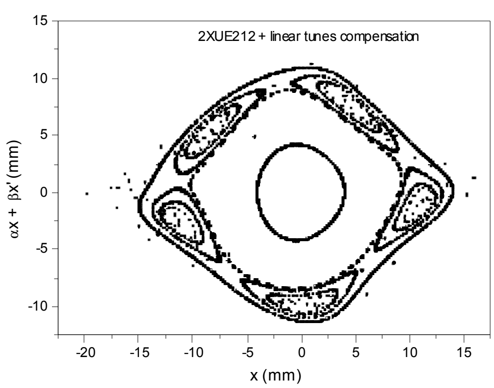


Fig.30 Normalised horizontal phase space showing excitation of unstable 5th order resonance ($5Q_x = 104$)

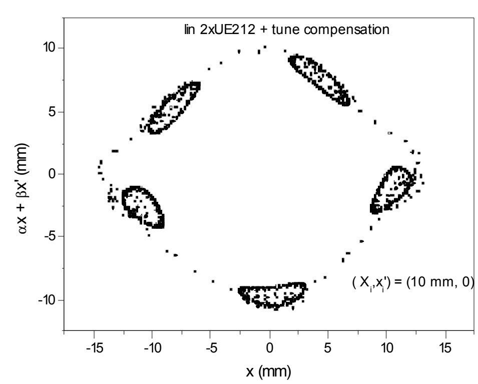


Fig.31 Normalised horizontal phase space showing excitation of unstable 5th order

with sextupoles + non-linear 2xUE212 + linear tunes sextupoles +
compensation.

resonance $5Q_x = 104$ with

linear 2xUE212 + linear tunes
compensation.

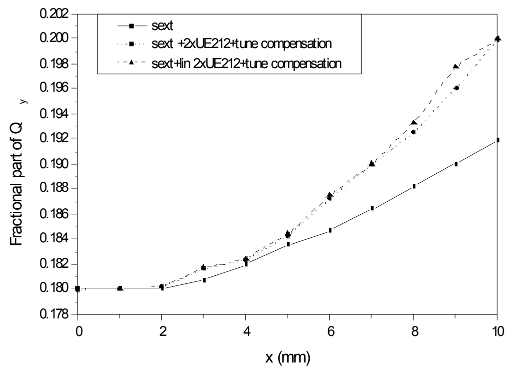


Fig.32 Horizontal amplitude-dependent tune with sextupoles, sextupoles + 2xUE212, sextupoles + linear 2xUE212 and linear tunes compensation in both cases.

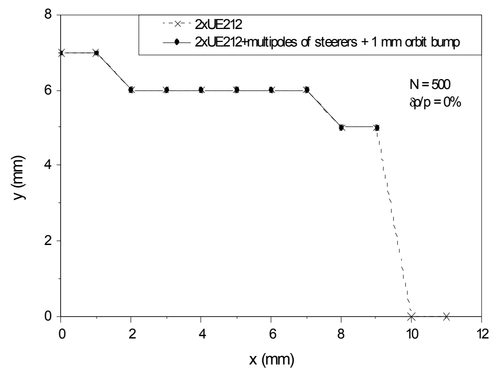


Fig.33 Dynamic aperture with orbit bump of 1mm and multipoles of chicane magnets with 2xUE212.

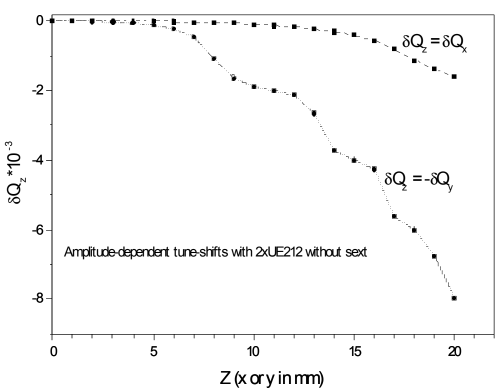


Fig.34 Amplitude-dependent tune-shifts with 2xUE212 without sextupoles

7.3 Wiggler (W61) :

The wiggler W61 is a high field horizontal planar device. It will be installed in 4m long short straight section 4S where -functions of machine are low ($_x$ = 1.14m, $_y$ = 1.6 m at middle of straight section) in order to minimize its influences on the beam dynamics. The simulation routine available in computer program RACETRACK based on polynomial field descriptions has been used. The parameters required for simulations are;

$B_0 = 2.05$ T , $b_2 = -1.069\ 10^2\ [m^{-2}]$, $c_3 = 1.45\ 10^6\ [m^{-4}]$, $d_4 = -\ 6.7\ 10^9\ [m^{-6}]$
------ (Vertical Poles)

Np = 33 , = 61 mm

Linear Distortions :

-asymmetries and Linear Tune-shifts :

The linear tune-shift and -asymmetries in horizontal plane are negligible. This device produces perturbations in vertical plane only. The -asymmetries (fig.35) are nominal (< 5%). Therefore, it is not required to correct the -asymmetries locally or globally. The vertical linear tune-shift is also small and is equal to 0.01.

Non-linear Effects :

To estimate the influences of non-linear fields, the vertical amplitude-dependent tune-shift (fig.42) has been calculated with device only (no sextupoles). Also, the both tune-shifts have been calculated with device in presence of sextupoles (fig.36 and fig.37). The horizontal tune-shift is nearly zero and vertical is small in region of our interest. Therefore, non-linear effects of this device are small and should not have severe effects on the dynamic aperture.

Dynamic Aperture :

The dynamic apertures have been calculated considering W61 as linear and non-linear devices with linear tunes compensation (fig.36). There is no significant reduction in it as a whole. It remains same in x while it is reduced by 1 mm in y. Also, there is no significant difference in two cases. It means that non-linear fields do not have any severe effects on dynamic aperture.

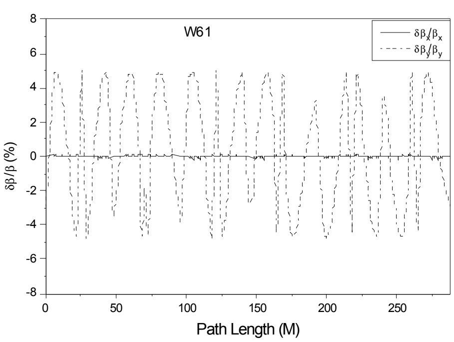


Fig. 35 -asymmetries around the ring with wiggler W61 without linear tunes compensation in both cases.

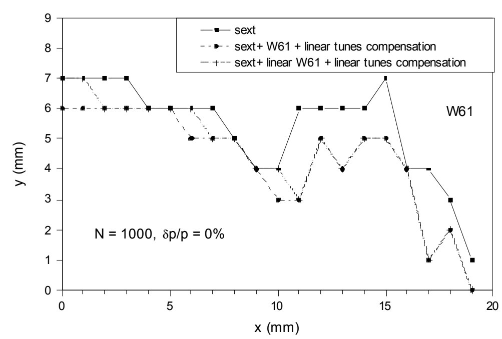
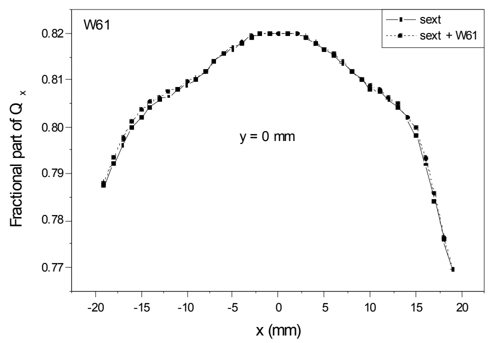


Fig.36 Dynamic aperture with sextupoles, sextupoles+ W61, sextupoles+ linear W61 (p/p = 0%) with linear tunes compensation in both cases.



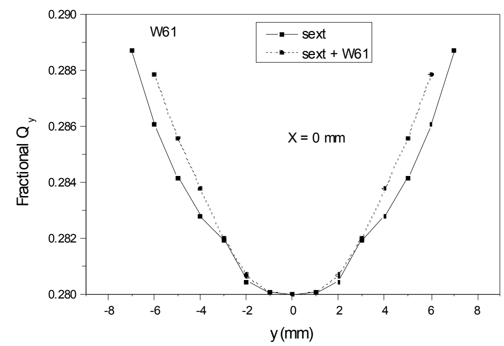


Fig.37 Horizontal amplitude-dependent tune with sextupoles, sextupoles + W61

Fig.38 Vertical amplitude-dependent tune with sextupoles, sextupoles + W61

The above parameters of device show that k_x is nearly zero. In order to investigate its effects when coupling is not zero. Further, studies have been carried out using simulation routine of RACETRACK for planar device based on ideal field descriptions for a planar device.

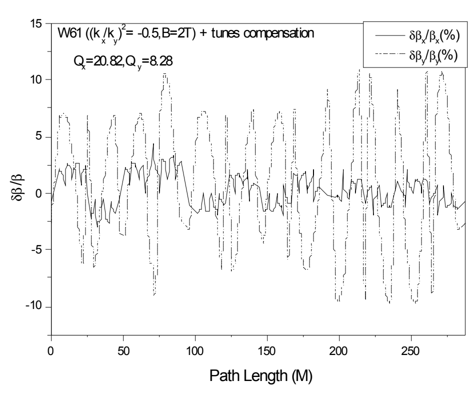
$(k_x/k_y)^2 = -0.5$

Linear tune shifts :

$Q_x = 0.008$, $Q_y = 0.018$

-asymmetries :

The -asymmetries (fig.38) in vertical plane are increased to nearly 10 % but has no major effect on dynamic aperture (fig.39) which can be estimated from difference of linear and non-linear cases. The dynamic aperture has reduced by 3 mm and 1 mm in x and y respectively.



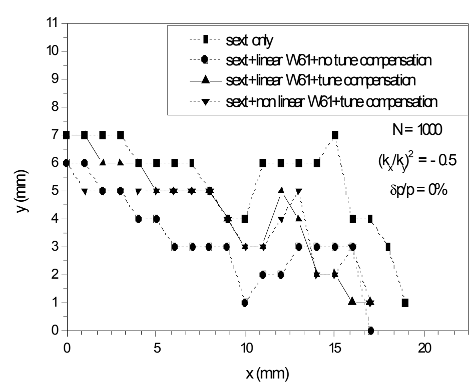


Fig. 39 -asymmetries with W61 ($(k_x/k_y)^2 = -0.5$) with linear tunes compensation

Fig.40 Dynamic aperture with W61($(k_x/k_y)^2 = -0.5$) + sextupoles

Therefore, it is concluded that W61 will have negligible effect on beam dynamics provided k_x is small. There is no need of compensation of -asymmetries of the ring in this case.

7.4 In-vacuum Undulator (U24) :

It is hybrid type and in-vacuum planar undulator and will be installed in short straight section 6S. A same simulation routine has been used as in case of W61. Its parameters required for simulations are listed below;

B_0 = 0.66 T , b_2 = -1.82 10^2 [m^{-2}] , c_3 = 1.99 10^6 [m^{-4}] , d_4 = -5.46 10^9 [m^{-6}]
----- (vertical poles)

Np = 80 , = 24 mm

Linear effects :

Q_x = 0.00 , Q_y = 0.001

The linear tune-shifts and -asymmetries are negligible. Therefore, its linear distortions can be ignored completely.

Non-Linear Effects :

The device is a planar device and k_x 0 for simulations parameters listed above. It will not have any affect in horizontal. The vertical amplitude-dependent tune-shift is small and is around 0.002 at y = 7 mm. Therefore, its non-linear effects are also negligible.

Dynamic Aperture :

The dynamic aperture in vertical plane shrinks from 7mm to 6 mm while no change in horizontal plane (fig.41).

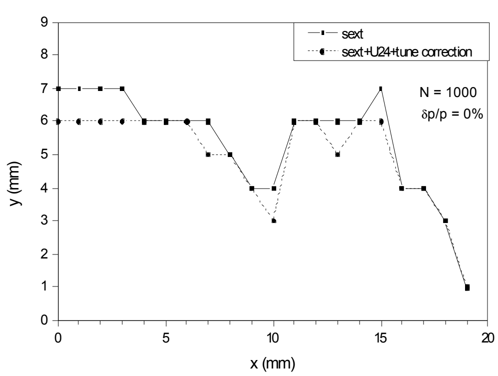


Fig. 41 Dynamic aperture with sextupoles and sextupoles + U24

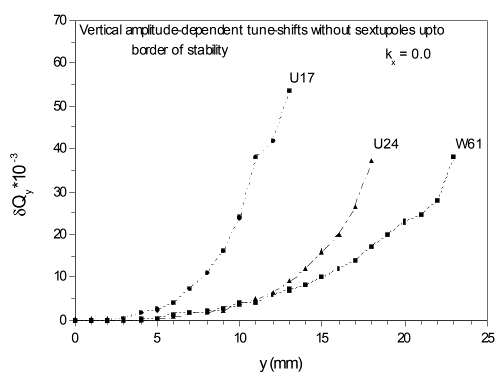


Fig.42 Vertical amplitude-dependent tune-shifts without sextupoles for planar devicesU17, U24, W61 for k_x = 0

The above parameters show that k_x is nearly zero. In order to investigate effects of coupling k_x, further studies have been carried out using simulation routine of RACETRACK for planar.

$(k_x/k_y)^2$ = -0.5

Linear tune shifts:

Q_x = 0.002 , Q_y = 0.004

-asymmetries :

($_{x,y}$/ $_{x,y}$)$_{max}$ < 3 %

The vertical dynamic aperture (fig.43) reduces further by 1 mm and but not much effect in horizontal. Therefore, strong coupling can limit dynamic aperture further.

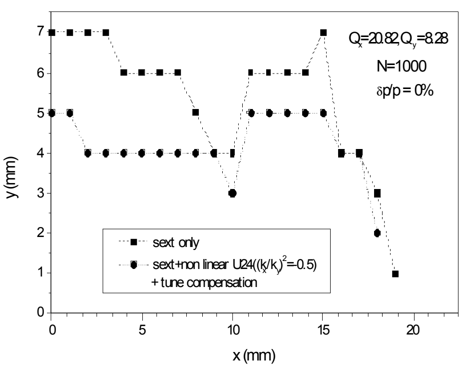


Fig.43 Dynamic aperture for sextupoles, sextupoles+ U24 ($(k_x/k_y)^2$ = -0.5) with linear tunes compensation

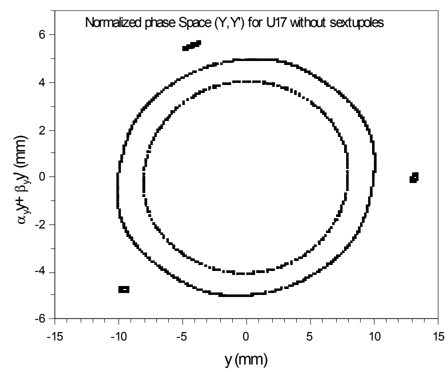


Fig.44 Normalized phase space upto border of stability for U17 without sextupoles (stability is limited by a sextupole resonance $3Q_y$ = 25).

7.5 *In-vacuum Undulator (U17) :*

It is hybrid type in-vaccum planar undulator. It will be installed in short straight section 6S in place of U24 in second phase. The simulation studies have been carried out using RACETRACK routine of ideal field descriptions of a planar device.

B_0 = 1.0 T, Np = 110 , = 17 mm

Linear tune-shifts :

Q_x = 0.0 , Q_y = 0.0021 --- for k_x = 0

Q_x = -0.0017 , Q_y = 0.0042 --- for $(k_x/k_y)^2$ = -0.5

-asymmetries :

($_{x,y}$/ $_{x,y}$)$_{max}$ negligible --- for k_x = 0

($_{x,y}$/ $_{x,y}$)$_{max}$ < 3% --- for $(k_x/k_y)^2$ = -0.5

The linear effects of this device are negligible but increase in vertical tune has to be corrected though it is nominal (already mentioned).

Non-linear effects :

The vertical amplitude-dependent tune-shift is maximum in case of U17 ($k_x = 0$) among three planar devices (fig.42). It is due to fact that non-linear effects are more severe in case of small period devices. It is in agreement with theory as non-linear terms in Hamiltonian are proportional to $^{-2}$. The vertical amplitude-dependent tune-shift is still less than 0.005 in a region of interest. The non-linear effects are still moderate.

Dynamic Aperture :

The vertical dynamics aperture (fig.45) is reduced by 2 mm in this case but remains nearly same in horizontal. There is not much effect of coupling on it.

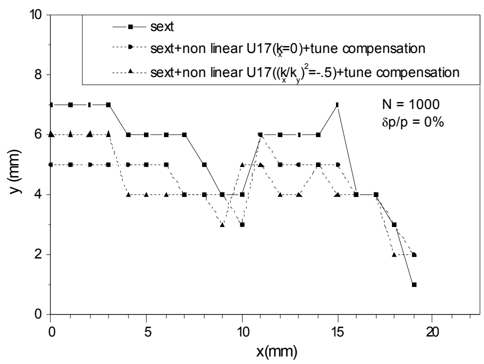


Fig. 45 Dynamic aperture for sextupoles, sextupoles+ U17 with $k_x = 0$ and $(k_x/k_y)^2 = -0.5$.

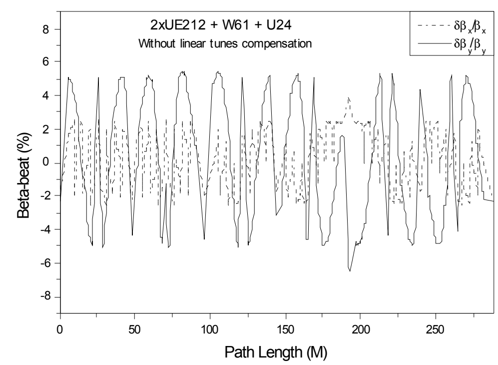


Fig.46 -asymmetries with 2xUE212+W61+U24

***Note :** During numerical investigations of non-linear behaviour of planar devices, it is noted that vertical stable amplitude (y = 12 mm) is limited due to excitation of sextupole resonance $3Q_y = 25$ (fig.44) in case of U17 (without ring sextupoles). According to the second order pertubation theory, this resonance can be excited by y^3 term in the Hamiltonian. In case of insertion device, it appear in oscillating sextupole-like terms. It has been already discussed in this report that these terms have very strong amplitude of oscillations and their averaged effect can not be ignored. But if matched end poles (orbit matching) are taken into account (under approximation that -functions of machine over period length are constant and variation of betatron tunes over it, are small) , the effects of these terms on beam dynamics are cancelled out. Therefore, the stability should not be limited by sextupole resonances. These observations leave a question yet to be understood that it happens either due to break of condition of cancellation or improper inclusion of end poles in simulation routine in RACETRACK for planar devices. In my opinion, the effects of end poles are not properly treated in simulation routine. In this case, numerical predictions are pessimistic.*

7.6 Combined Effects (2xUE212 + W61 + U24) :

In order to study the combined effects of all IDs on beam dynamics, three devices 2xUE212, W61 and U24 are considered together. The simulation routine based on polynomial field descriptions of RACETRACK is used for numerical investigations for all devices. The APPLE-II (2xUE56) could not be included due to unavailability of simulation routines required for 2XUE212 and 2xUE56 in one computer code. Anyway, effects of 2xUE56 are small and can be ignored on the beam dynamics.

Linear Perturbations :
Linear tune-shifts :
Q_x = -0.0093 , Q_y = 0.0273
-asymmetries :
$(\;_x / \;_x)_{max}$ < 4 %
$(\;_y / \;_y)_{max}$ < 6 %
(from fig.45)
The maximum -asymmetries are still nominal and less than 6%. Therefore, there is no need of any correction of -functions. The dynamic aperture in vertical plane (fig.47) improves by 1 mm when linear tunes are not compensated. It is due to sensitivity of dynamic aperture with increase in vertical tune. It is seen from fig.49 that it decreases first till Q_y = 8.292 and then improves again. This is the reason that it is better than in case of no IDs. Also, horizontal dynamic aperture is improved by 5 mm in comparison to a case when linear tunes are compensated. It is due to more equilibrium -asymmetries around the ring.

Effect of Shifting of Operating Point on Dynamic Aperture :

The horizontal dynamic aperture drops due to excitation of $5Q_x$ = 104 by 2xUE212 when linear tunes are compensated. The fractional horizontal tune approaches to 0.8 from 0.82. In order to avoid excitation of this resonance as well as sensitivity to increase in Q_y, the operating point is shifted to (Q_x = 20.85, Q_y = 8.27) and it shows improvement in horizontal and vertical dynamic aperture (fig.48).

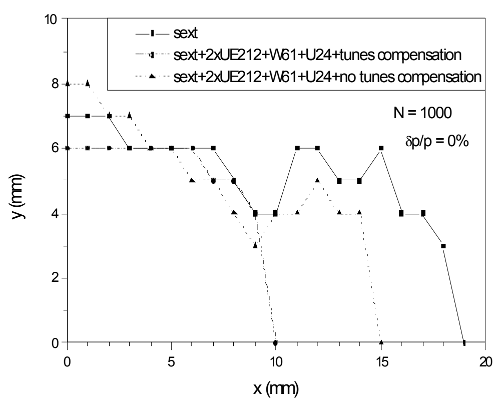


Fig.47 Dynamic aperture with sextupoles+2xUE212 +W61+U24 with and without linear tunes compensation.

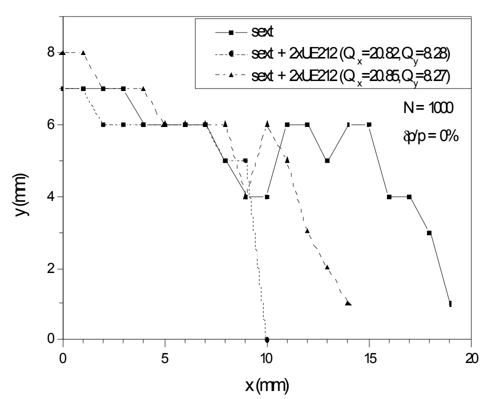


Fig.48 Dynamic aperture after shifting linear tunes Q_x = 20.82 to Q_x = 20.85 and Q_y = 8.28 to 8.27.

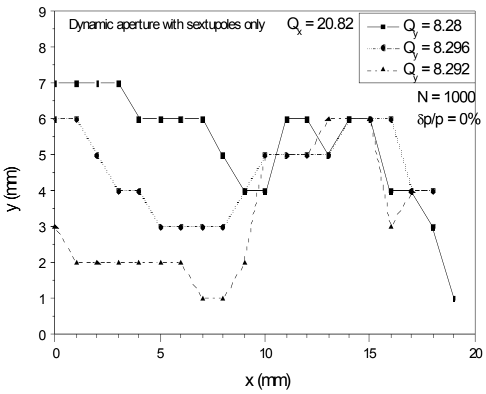


Fig.49 Dynamic aperture with sextupoles only for different Q_y keeping $Q_x = 20.82$.

8. Summary and Conclusions :

The various numerical models developed at BESSY, ELETTRA and ESRF for simulations studies of different types insertion devices have been studied to select proper routines to simulate and get required input parameters for SLS devices. The model developed at BESSY by G. Wüestefeld et al. for APPLE II/Sasaki type device and implemented in computer program BETA, has been used to study 2xUE56 device. A numerical model developed by L. Tosi et al. at ELETTRA in which the transverse fields are expressed in form of polynomial of sixth order and coefficients are obtained from either measured or numerical fields data. It is implemented in computer program RACETRACK. It has been used for simulation studies of 2xUE212, W61, U24 and U17. Also, the simulation routine based on ideal planar field descriptions of RACETRACK, has been used to study effects coupling (k_x 0) for planar devices W61,U24 and U17.
The simulations studies have been carried out with different IDs and its findings are briefed as follows;

The dynamic aperture (in case of sextupoles only) in vertical plane is sensitive to a small increase in vertical tune Q_y. Firstly, it decreases till $Q_y = 8.293$ and start recovering afterward keeping strengths of sextupoles as constant. Due to this reason, the dynamic aperture calculations in presence of IDs show sharp reduction in vertical plane in a case vertical linear tune is not corrected. It occurs due to their quadrupole-like effects. Therefore, the dynamic aperture has to be calculated with compensations of linear tunes for individual cases.

a. APPLE II/Sasaki-type (2xUE56) :

i. The maximum -asymmetries and linear tune-shifts are less than 4% and 0.005 respectively for entire range of shift parameter. The linear perturbations produced by this device are nominal.

ii. A small reduction in dynamic aperture in vertical plane is noticed when increase in vertical linear tune is not compensated. It recovers after compensation. While no significant effect is observed in horizontal plane.

iii. The non-linear effects of this device are estimated by amplitude-dependent tune-shifts and distortions of phase space trajectories in absence of sextupoles. The maximum amplitude-dependent tune-shifts produced in the region of interest for SLS are;

$Q_x < 10^{-4}$ ----- for x = 20 mm

$Q_y < 1.5\ 10^{-3}$ ----- for y = 7 mm

The normalized phase space plots show that trajectories in this region are perfect circle and amplitude-dependent tune-shifts are negligible. Therefore, non-linear effects of this device can be neglected for entire range of shift parameter.

iv. This device will not have any significant effect on dynamic aperture for entire range of shift parameters provided vertical linear tune is compensated.

v. There is no effects of parallel displacement of beam and multipoles of chicane magnets on the dynamic aperture though side magnets have significant sextupole component. But their effects are cancelled out due to alternate polarities.

b. Elliptic Electromagnetic Wiggler (2xUE212) :

i. The linear perturbations are;

Linear tune-shifts :

Qx = - 0.0091 , Qy = 0.0166

-asymmetries :

< 4 %	------	Without compensation of linear tune-shifts
< 6%	------	With compensation of linear tune-shifts (Except at the middle of device)

Though -asymmetries are nominal but are increase when linear tune-shifts are compensated.

ii. The dynamic aperture in horizontal plane shrinks from 19 mm to 10 mm due to linear perturbations. This occurs due to excitation of intrinsic 5th order sextupole resonance $5Q_x = 104$. It appears due to loss of symmetry. The fractional horizontal tune reduces from 0.82 to 0.8 due to change in -functions at the sextupole locations and betatron phases between them and it changes the dependence of tune with amplitude for sextupoles.

iii. No significant effect is observed on the dynamic aperture in vertical plane provided increase in a vertical tune is compensated .

iv. The amplitude-dependent tune-shifts due to non-linear field of device are less than10^{-3} in a region of interest. Therefore, non-linear effects of this device are negligible.

v. No effects of beam displacement and multipoles of chicane magnets are observed on the dynamic aperture.

c. Wiggler W61:

i. The linear perturbations are;

Linear Tune-shifts :

Qx = 0.0002 , Qy = 0.01

-asymmetries :

< 5 % ---- vertical plane (with compensation of linear tune-shift)

ii. No significant effect is observed on dynamic aperture when vertical linear tune is compensated. A reduction of 1 mm occurs in vertical plane while no change in horizontal.

iii. The amplitude-dependent tune-shift is produced mainly in a vertical plane and is less than 2.10^{-3} in a region of interest. Therefore, its non-linear effects are feeble and will not have any significant effect on the dynamic aperture.

iv. Input parameters provided by ID group indicate that k_x 0 . But strong value of it, can change these conclusions.

v. In-vacuum Undulator U24 :

i. The linear perturbations are negligible and will not have any impact on beam dynamics.

Linear tune-shifts :

Qx = 0.00 , Qy = 0.001

-asymmetries : negligible

ii. The amplitude-dependent tune-shift is produced mainly in a vertical plane and is roughly same as in case of W61 in region of our interest though it will be more in comparison to W61 for higher amplitude due to smaller period length.

iii. No significant effect on dynamic aperture is observed. The reduction of 1 mm occurs in vertical plane while remains unchanged in horizontal.

iv. The input parameters provided by ID group indicate that k_x 0. In this case, above conclusions can change.

e. In-vacuum Undulator U17:

i. The linear perturbations are negligible in this case also (k_x 0) and hence will not have any impact on beam dynamics .

Linear tune-shifts :

Qx = 0.00 , Qy = 0.002

-asymmetries : negligible

ii. This device produces the strongest non-linear effects among three planar devices due to shortest period length. The vertical amplitude-dependent tune-shift is around 5.10^{-3} at y = 7 mm.

iii. The dynamic aperture in vertical plane reduces by 2 mm while by 1 mm in horizontal.

f. Combined Effects (2xU212+W61+U24) :

i. The linear perturbations are;

Linear tune-shifts :

Qx = - 0.0093 , Qy = 0.0273

-asymmetries :

< 4 %	------	horizontal
< 6 %	------	vertical

These are still nominal and no increase in it due to many devices.

ii. The dynamic aperture is improved in both planes if linear tunes are not compensated. It improves in horizontal plane from 10 mm to 15 mm though it was 10 mm in case of 2xUE212 only. It is 8 mm in vertical while it was 6 mm in case of planar devices. It is limited to 9 mm in horizontal if linear tunes are compensated. It happens due more equilibrium -asymmetries and shift of fractional vertical tune above 0.3.

iii. The horizontal dynamic aperture shows improvement in case operating point is shifted in such a way that fractional horizontal tune moves away from 0.8.

> Therefore, It is concluded that all devices can be operated in the ring. The dynamic aperture is enough in presence of all devices for efficient injection and good beam life time (without compensations of linear tune-shifts). Though, APPLE-II device is not included in studies of combined effects due to problem of unavailability of all simulation tools in one computer code. It has not much impact on beam dynamics. Also, its -asymmetries will try to make it more favourable.

Acknowledgements :

The authors will like to pay their sincere thanks to Dr. G. Wüestefeld of BESSY-II, Berlin for providing BESSY version of computer code BETA and valuable discussions and suggestions during course of these studies. Also, We thank to Dr. G. Ingold and Dr. T. Schimdt of ID group of SLS for providing input parameters required for simulations studies and discussions and suggestions. We also thank to Dr. M.Böge of beam dynamics group at SLS for introducing the computing environment.

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