

## A general top-up equation

### Introduction:

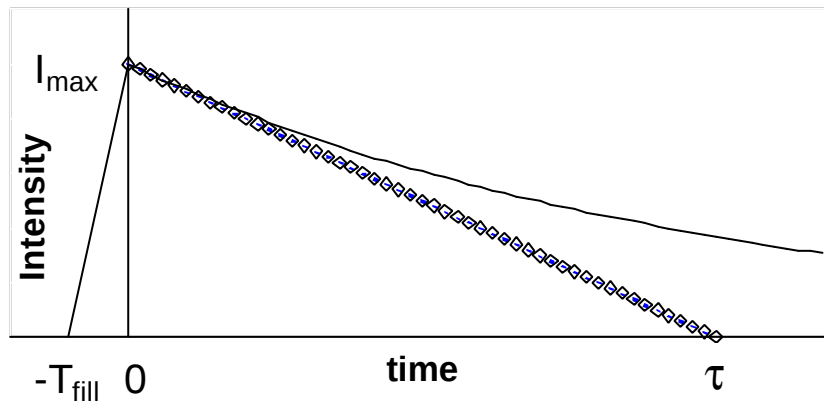
Electrons coming from the Linac are accelerated by the Booster synchrotron to the fixed energy of the storage ring (SR), where they are accumulated. The maximum intensity  $I_{\max}$  and the maximum charge  $Q_{\max}$  accumulated in the SR are related by  $I_{\max} = Q_{\max} / T_0$ , where  $T_0$  is the revolution time in the SR ( $= 0.96 \mu\text{s}$  for SLS).

### Normal filling:

The Booster has an acceleration cycle of period length  $T_B$ . The SR is filled from zero to the maximum intensity  $I_{\max}$  with an average charge  $Q$  injected every booster cycle. The time it takes to do this filling is  $T_{\text{fill}}$ . Since we have  $Q_{\max} / Q = T_{\text{fill}} / T_B$  we get an **average filling rate**:

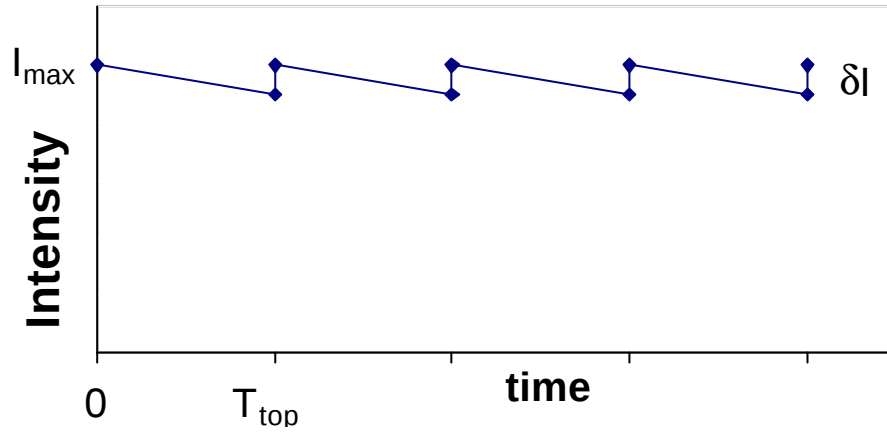
$$dI / dt = I_{\max} / T_{\text{fill}} = Q T_0 / T_B \quad (1)$$

After the filling procedure the intensity decays exponentially with a lifetime constant  $\tau$ .



### Top-up Injection:

However it is advantageous to keep the beam intensity in the SR practically constant with the so called **top-up** injection mode: We inject periodically a charge  $Q_{\text{top}}$  from a single Booster cycle into the SR with a time interval  $T_{\text{top}}$  between consecutive top-up shots.



Using a single cycle has the advantage, that the disturbance of the circulating beam by the injection process is damped out after about 20 ms. During this time interval an experimenter may be able to gate out the data taking process. On the other hand using several cycles to top up the intensity would disturb the stored beam for several seconds.

Top-up injection into the SR gives an equilibrium between the decay and the refilling process, leading to the relation

$$Q_{\text{max}} / Q_{\text{top}} = \tau / T_{\text{top}} \quad (2)$$

We have thus in total four characteristic time values:

1.  $T_B$  = Booster cycle ( = 0.32 s in the case of SLS)
2.  $T_{\text{fill}}$  = time to fill the SR from 0 to  $I_{\text{max}}$
3.  $\tau$  = lifetime
4.  $T_{\text{top}}$  = time interval between top-up shots

These times are connected with a general “**top-up equation**”:

$$T_{\text{top}} T_{\text{fill}} = f T_B \tau \quad (3)$$

where we have introduced the fractional charge  $f \equiv Q_{\text{top}} / Q$ . The intensity is not quite constant, but has very small fluctuations given by:

$$\delta I / I = Q_{\text{top}} / Q_{\text{max}} \quad (4)$$

Usually top-up injection occurs with the same charge as for regular injection ( $Q_{\text{top}} = Q$ ,  $f = 1$ ). We get then an even simpler equation, which does not contain explicitly the charge  $Q$  nor the maximum intensity  $I_{\text{max}}$  !

$$T_{\text{top}} T_{\text{fill}} = T_B \tau \quad (5)$$

### Example for SLS:

$$I_{\text{max}} = 300 \text{ mA}, \rightarrow Q_{\text{max}} \approx 300 \text{ nC}$$

$$\tau \approx 9 \text{ h} = 32'000 \text{ s}$$

$$Q_{\text{top}} = Q = 0.2 \text{ nC} \quad (f = 1)$$

$$T_{\text{fill}} = T_B Q_{\text{max}} / Q = 0.32 \text{ s} \cdot 300 \text{ nC} / 0.2 \text{ nC} = 480 \text{ s} = 8 \text{ min}$$

$$dI / dt = I_{\text{max}} / T_{\text{fill}} = 300 \text{ mA} / 480 \text{ s} = 0.6 \text{ mA/s}$$

$$T_{\text{top}} = \tau T_B / T_{\text{fill}} = 32'000 \text{ s} \cdot 0.32 \text{ s} / 480 \text{ s} = 20 \text{ s}$$

$$\delta I / I = Q_{\text{top}} / Q_{\text{max}} = 0.2 \text{ nC} / 300 \text{ nC} = 7 \cdot 10^{-4}$$

The intensity fluctuations are thus much smaller than usually required by experiments. One could then have the idea to inject less frequently than every 20 s. If one increases e.g. the top-up period to 5 min. the intensity drop between two refills is still as low as 1%. However to get the required charge of 3 nC we have to inject for 15 booster cycles or 5 s ,with the correspondingly longer perturbation of the stored beam, as mentioned before.