PAUL SCHERRER INSTITUT

SWISS LIGHT SOURCE

SLS-TME-TA-2002-0193
27th May 2005

SLS booster-to-ring transferline optics for optimum injection efficiency

Andreas Streun

Paul Scherrer Institut, CH-5232 Villigen PSI, Switzerland

Introduction

The original design of the booster-to-ring (BR) transferline provided matching of the beam from the booster to the storage ring, assuming identical optical parameters of injected and stored beam at the location of the injection septum [1]. However, considering the storage ring's minimum acceptance required to completely catch the injected beam, this is not the optimum. In the following, we will calculate the optimum matching in general, apply it to SLS, establish the corresponding BR-optics and report on the improvements thus achieved at SLS.

Optimum injection matching

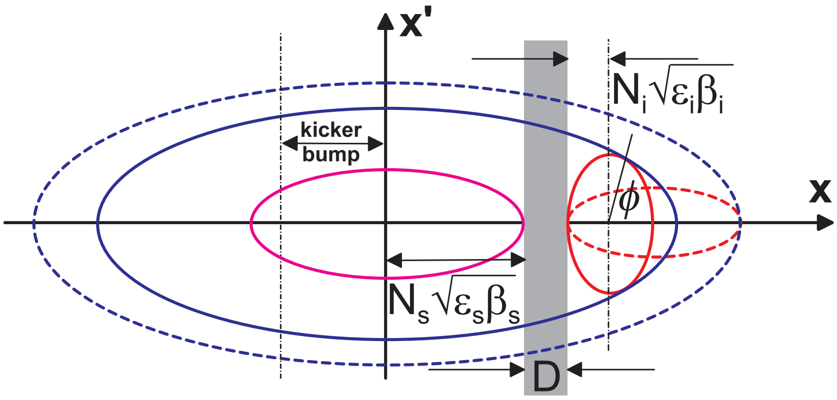

Figure 1: Injection phase space: Stored beam displaced by kicker bump (purple), injected beam (red) for original (dotted) and optimum matching (solid) and corresponding acceptances (blue) required to completely catch the injected beam. In phase space the septum appears as strip of width D (grey).

Figure 1 shows the injection into the storage ring in phase space (x,x'), since injection is done horizontally in SLS: ϵ_s and ϵ_i define stored and injected beams' emittances, β_s and β_i are the betafunctions at injection. The injected beam from the booster requires an acceptance A of the storage ring in order to be captured up to N_i standard deviations. The distance of the stored beam to the septum of thickness D is determined by another number N_s of stored beam standard deviations required to avoid too large losses at the septum.

In the following, the acceptance will be calculated and minimized in respect to the injected beam's properties. As already sketched in the figure, the required acceptance for $\beta_i < \beta_s$ (solid) will be smaller than the acceptance for $\beta_i = \beta_s$ (dotted).

For simplicity we consider only the case that both stored and injected beams have foci at the septum, i.e. $\alpha_s = \alpha_i = 0$, and that there is no dispersion. This assumptions are true for SLS. We further have to restrict the calculations to linear transformations.

With these simplifications and using the stored beam's center as origin (the displacement due to the injection bump is irrelevant for our considerations), the acceptance ellipse is described by

$$\frac{1}{\beta_s}x^2 + \beta_s x'^2 = A \tag{1}$$

The ellipse enclosing N_i standard deviations of the injected beam is described by

$$\frac{1}{\beta_i}\left(x - \left[N_s\sqrt{\epsilon_s\beta_s} + D + N_i\sqrt{\epsilon_i\beta_i}\right]\right)^2 + \beta_i x'^2 = N_i^2\epsilon_i \tag{2}$$

Now the acceptance can be calculated by elimination of x'^2 and requesting a unique solution for x. It is however more instructive to calculate it by differentiation: If we consider a particle on the perimeter of the injected beam's N_i-sigma ellipse, its coordinates are given by

$$x = N_s\sqrt{\epsilon_s\beta_s} + D + N_i\sqrt{\epsilon_i\beta_i}(1+\cos\phi) \qquad x' = N_i\sqrt{\frac{\epsilon_i}{\beta_i}}\sin\phi \tag{3}$$

As visible in figure 1, the particle coordinate x relative to the stored beam's center contains contributions from the stored beam stay-clear, the septum thickness, the injected beam stay-clear and the angle ϕ [1]. Inserting eq.3 into the acceptance from eq.1 and, for convenience, introducing the abbreviations

$$a := N_i^2\epsilon_i \qquad b := \sqrt{\frac{\beta_i}{\beta_s}} \qquad \delta := \frac{N_s\sqrt{\epsilon_s\beta_s} + D}{N_i\sqrt{\epsilon_i\beta_s}}$$

the acceptance can be written as

$$A = \frac{a}{b^2}\left(1 + (\delta+b)^2 + 2b^3(\delta+b)\cos\phi - (1-b^4)\cos^2\phi\right). \tag{4}$$

In order to accept the complete injected beam, the maximum of A with respect to ϕ has to be found:

$$\frac{\partial A}{\partial\phi} = 0 \quad \longrightarrow \quad \sin\phi\left(b^3(\delta+b) - (1-b^4)\cos\phi\right) = 0$$

The trivial solution is given for $\phi = 0°$. A more interesting solution is given by

$$\cos\phi = \frac{b^3(\delta+b)}{1-b^4}. \tag{5}$$

In the case of an upright ellipse, corresponding to an extremely focussed injected beam ($b\to 0$) the angles $\phi = \pm 90°$ determine the maximum acceptance.

Since both δ and b are positive by definition, solutions of eq.5 exist only for

$$0 < b \leq b_{\text{lim}}, \qquad \text{with} \qquad 2b_{\text{lim}}^4 + \delta b_{\text{lim}}^3 = 1. \tag{6}$$

As long as the local curvature of the injected ellipse is smaller than the curvature of the acceptance ellipse, $\phi = 0°$ defines the acceptance. If the curvature becomes smaller for $b < b_{\text{lim}}$ from eq. 6, two new tangential points with $\pm\phi$ given by eq.5 define the acceptance

[1] The angle ϕ in the drawing is only correctly shown, if the injected beam's ellipse happens to be a circle.

and $\phi = 0°$ becomes a local minimum as easily verified by evaluating $\partial^2 A/\partial\phi^2$, also see figure 1.

By introducing eq.5 into eq.4 the acceptance for $0 < b \leq b_{\mathrm{lim}}$ can be written as

$$A = a\left(\frac{1}{b^2} + \frac{(\delta + b)^2}{1 - b^4}\right) \tag{7}$$

The acceptance for $b \geq b_{\mathrm{lim}}$ is simply given by

$$A = a(\delta + 2b)^2 \tag{8}$$

In order to find the optimum matching of the injected beam requiring the minimum acceptance for full capture, we have to minimize A with respect to b. This gives the unwieldy expression

$$\frac{\partial A}{\partial b} = 0 \quad \longrightarrow \quad 3\delta b^7 + 2\delta^2 b^6 + 3b^4 + \delta b^3 = 1 \tag{9}$$

However, we can introduce eq.5 into this equation by careful bracketing and suitably adding zeros. After some algebra we thus arrive at a simple quadratic equation for the angle of the tangential points:

$$2\cos^2\phi + \cos\phi = 1 \tag{10}$$

The two solutions $\cos\phi = -1$ and $\cos\phi = 1/2$ correspond to $\phi = 180°$ and $\phi = \pm 60°$. For $\phi = 180°$ the injected beam would be completely outside the acceptance, which is of no interest. The minimum acceptance for capture of the whole injected beam thus is given for $\phi = \pm 60°$. This value is independant of the particular choice of δ and b! Only the requirement from eq.6 has to be fulfilled for this solution to exist.

Introducing this solution into eq.5 the equation to obtain the optimum b is simplified to

$$3b_{\mathrm{opt}}^4 + 2\delta b_{\mathrm{opt}}^3 = 1. \tag{11}$$

However, this equation still has to be solved numerically for the specific value of δ.

Application to the SLS BR transferline

Beam parameters for the booster at extraction, for the storage ring at injection and for the optimum injection matching are given in the table below:

<table>
<tr><td rowspan="2">Location</td><td>booster extraction</td><td>stored beam</td><td>optimum injection</td></tr>
<tr><td>ABOMA-BD-6D exit</td><td colspan="2">ARIMA-YIN exit</td></tr>
<tr><td>Horizontal emittance ϵ_x [mm·mrad]</td><td>0.01</td><td>0.005</td><td>0.01</td></tr>
<tr><td>Horizontal beta function β_x (= β) [m]</td><td>2.68</td><td>4.71</td><td>0.64</td></tr>
<tr><td>Horizontal alpha function α_x [rad]</td><td>−0.61</td><td>-0.18</td><td>≈0</td></tr>
<tr><td>Vertical beta function β_y [m]</td><td>9.92</td><td colspan="2">3.90</td></tr>
<tr><td>Vertical alpha function α_y [rad]</td><td>2.68</td><td colspan="2">-0.22</td></tr>
<tr><td>Horizontal dispersion η_x [m]</td><td>0.22</td><td colspan="2">0</td></tr>
<tr><td>Horizontal dispersion's slope η'_x [rad]</td><td>0.10</td><td colspan="2">0</td></tr>
</table>

The booster values are from the `B39` optics at a working point of 12.40/8.38, close to the original design [2], for providing smallest beam diameters. Recently, by shaping of quadrupole waveforms, constant tunes over the ramp have been achieved in order to ensure the validity of the optical parameters also at extraction. Measurements of beta functions at injection energy had proven agreement with design [3].

For the ring, the standard `D2R` optics for low emittance with dispersion free straight was used, with the working point at 20.38/8.16. The betafunctions have been measured and corrected to agree with the design to a few percent [4].

The optimum injection parameters are based on matching the BR line to the solution of eq.11, with the δ-parameter determined by

$$N_s = 20, \qquad N_i = 3, \qquad D = 3 \text{ mm},$$

with the numbers of stored and injected beam sigmas arbitrarily chosen and D the thickness of the septum. The limiting and optimum values for b and β_i from eqs. 6, 11 thus are

$$b_{\rm opt} = 0.37 \longrightarrow \beta_{i,\rm opt} = 64 \text{ cm}, \qquad b_{\rm lim} = 0.46 \longrightarrow \beta_{i,\rm lim} = 1.0 \text{ m}.$$

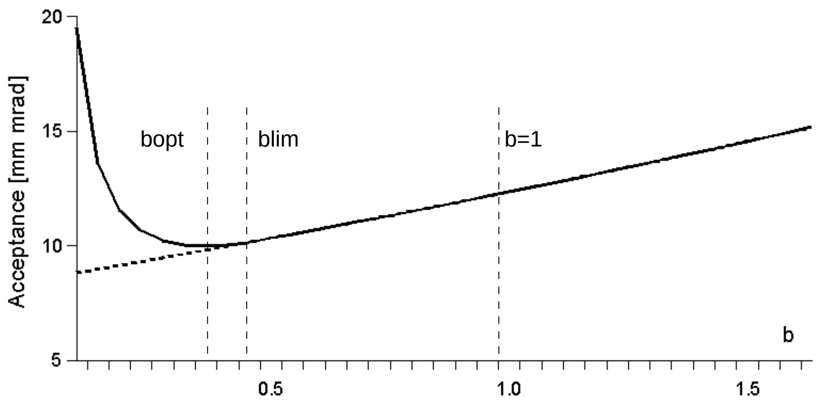

Figure 2: Acceptance vs. b-parameter for SLS injection: The solid line corresponds to eqs.7 and 8 over the whole range of b-values, the dotted line is eq.8 for $b < b_{\rm lim}$

The horizontal acceptance A of the SLS storage ring as a function of b is shown in figure 2: Changing the injection from the current $b = 1$ setting to $b = b_{\rm opt}$ reduces the required acceptance from 11.7 to 9.4 mm·mrad, resp. the required [dynamic] aperture (acceptance projected onto x-axis) from 7.3 to 6.5 mm. Although this improvement seems small, the gain in injection efficiency is significant if injection operates at the very edge of dynamic aperture.

Figure 3 shows the optics of the booster to ring transferline: Matching to the exact optimum value of $\beta_{x,i,\rm opt} = 64$ cm inverted the polarities of `ABRMA-QB-1` and `QC-2`, however, since the values were small, both quadrupoles could be set to zero. The matching with the remaining five quadrupoles and $\beta_{x,i}$ as a free variable resulted in a value of $\beta_{x,i,\rm opt} = 68$ cm. Considering the weak dependancy of acceptance on b as shown in figure 2, this result certainly is good enough.

The table gives the required quadrupole currents at 2.4 GeV:

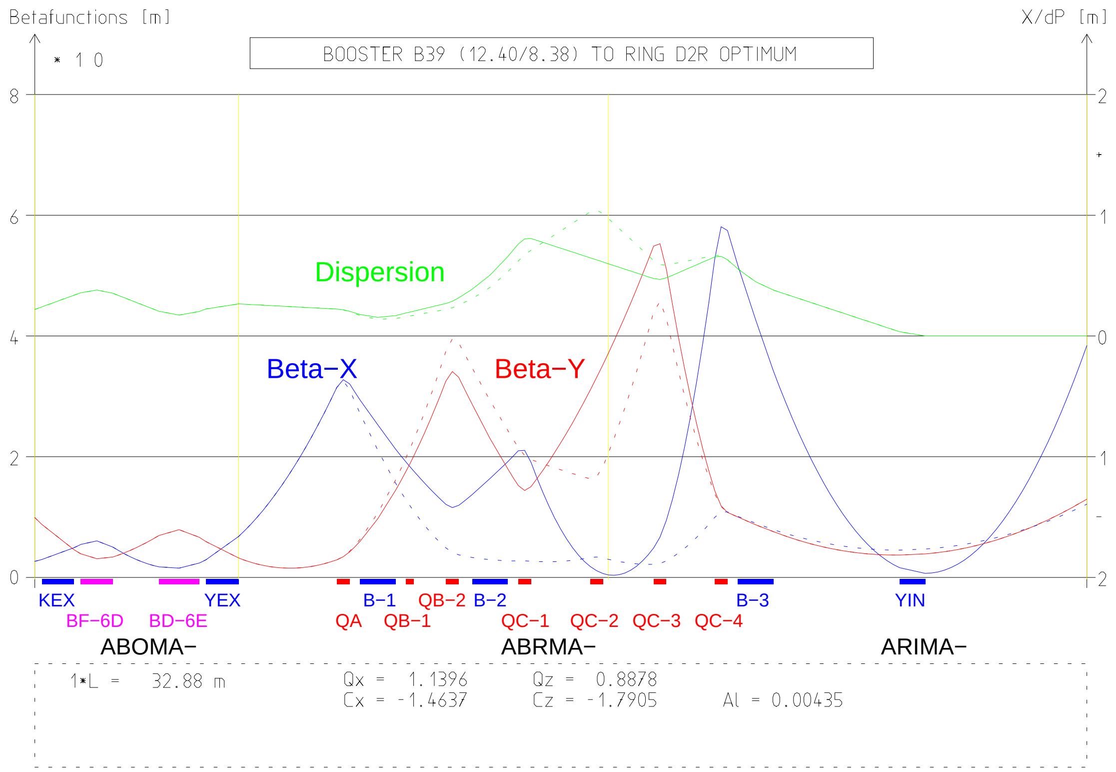

Figure 3: BR transferline optics for old $b = 1$ matching (dotted) and new $b \approx b_{\text{opt}}$ matching (solid). The line as shown starts with the extraction kicker in the booster and ends at the end of the ring injection straight.

ABRMA-	QA	QB-1	QB-2	QC-1	QC-2	QC-3	QC-4
I [A]	53.4	0	65.6	88.1	0	74.8	92.4

Results

Current in the booster $I_{\text{bo}}(t)$ is measured by the MPCT and shown on a scope in the control room. Current increase per second in the ring $\dot{I}_{\text{ri}}$ is obtained from the ring PCT current measurement. The injection efficiency is the ratio of charge accepted by the storage ring over the charge in the last booster turns before extraction:

$$\text{Injection efficiency} = \frac{\Delta q_{\text{ri}}}{q_{\text{bo}}} = \frac{\dot{I}_{\text{ri}} C_{\text{ri}}}{I_{\text{bo}}(t_{\text{ext.}}) C_{\text{bo}} f_{\text{rep}}} = 0.3413 \frac{\dot{I}_{\text{ri}}[\text{mA/s}]}{I_{\text{bo}}(t_{\text{ext.}})[\text{mA}]}$$

with $C_{\text{ri}} = 288$ m, $C_{\text{bo}} = 270$ m ring and booster circumferences, and $f_{\text{rep}} = 3.125$ Hz the injection repetition frequency.

With the original transferline setting, efficiencies of up to 100 % could be achieved with nominal ring chromaticities $\xi_x = \xi_y = +1$, however with increased chromaticities of $\xi_x = \xi_y \approx +5$, as required for suppression of the coupled bunch instabilities at higher current, the best efficiency values ever reached amounted to ≈50 %.

With the optimum transferline setting, the efficiency at nominal chromaticities was 100 % without much fine tuning, and up to 80 % at large chromaticities of $\xi_x = \xi_y = +6$.

The remaining losses of 20 % are probably due to an energy mismatch between booster and ring affecting the injected beam stronger at high chromaticities and subject to further investigations.

Furthermore, design calculations predicted an horizontal storage ring acceptance of $\approx$ 30 mm·mrad, which is much larger than the effective acceptances of about 10 mm·mrad as encountered in the injection process. This discrepancy has to be explored.

References

[1] The SLS Design Handbook, sec. 3.4.2

[2] W. Joho et al., *The SLS booster synchrotron*, EPAC'98, Stockholm, June 1998

[3] W. Joho, *SLS booster at its design performance*, PSI Scientific Report 2000, Vol. VII

[4] M. Böge, A. Streun, *Measurement and compensation of linear optical distortions*, PSI Scientific Report 2001, Vol. VII

Injection with dispersion

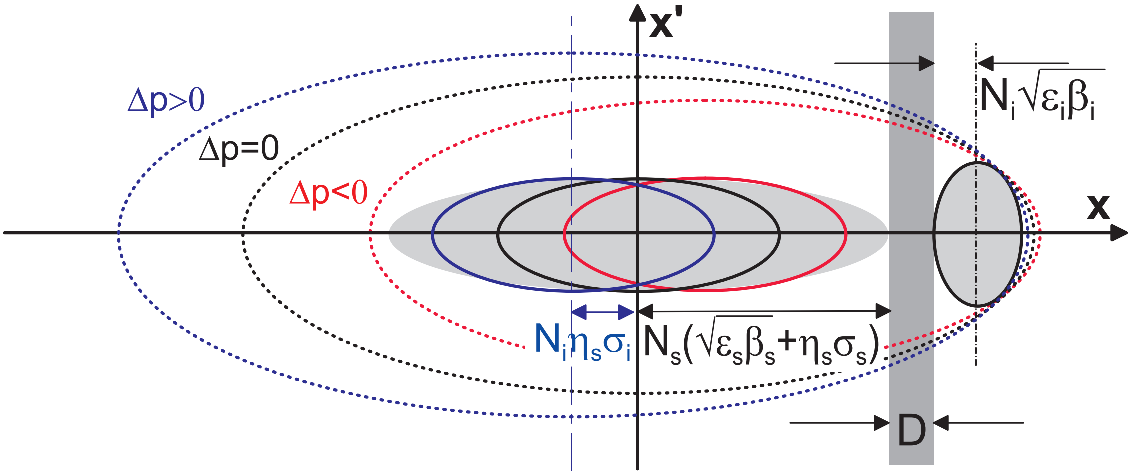

Figure 4: Injection at a dispersive location: The beam ellipse (black) appears shifted for momentum deviations (red and blue), consequently the acceptances to capture the satellite beam depend on momentum too. (The signs of momentum deviations as arbitrarily sketched here depend on the sign of the dispersion function and whether injection is done from the ring inside or outside.)

If the stored beam has non zero dispersion η_s at injection, some of the equations above have to be modified: As shown in figure 4 (in comparison to fig.1), the required N_s-sigma distance from the septum is now given by the radius of the grey ellipse, obtained from the convolution of the natural beam emittance with the widening by local dispersion:

$$N_s\sqrt{\epsilon_s\beta_s + (\eta_s\sigma_s)^2},$$

where σ_s indicates the stored beam's rms relative energy spread.

The injected beam must not have any dispersion, because any dispersion could only widen it, as obvious from figure 4: Thus $\eta_i = 0$ is the optimum value, no matter what is the dispersion of the stored beam, η_s .

The acceptance for capture of the satellite beam depends on momentum and is largest for the left ellipse in figure 4. A particle on the perimeter of the injected beam's N_i-sigma ellipse relative to the origin of this stored beam's off momentum ellipse thus is given by

$$x = N_s\sqrt{\epsilon_s\beta_s + (\eta_s\sigma_s)^2} + D + N_i\eta_s\sigma_i + N_i\sqrt{\epsilon_i\beta_i}(1+\cos\phi) \qquad x' = N_i\sqrt{\frac{\epsilon_i}{\beta_i}}\sin\phi$$

Here we added the distance between the on- and off-momentum stored beam centres, where the relevant momentum deviation was identified by the requirement to also capture N_i sigma of the injected beam's energy distribution of rms value σ_i.

Comparison to eq.3 tells that the calculation may proceed in the same way as before, by simply replacing the quantity δ by

$$\tilde{\delta} := \frac{N_s\sqrt{\epsilon_s\beta_s + (\eta_s\sigma_s)^2} + D + N_i\eta_s\sigma_i}{N_i\sqrt{\epsilon_i\beta_s}}$$

Consquently we will obtain another result for the optimum betafunction β_i of the injected beam as in the non-dispersive case.

Aperture requirements

In order to save kicker strength one would like to place the septum close to the optical axis. On the other hand, the distance has to be chosen sufficiently large in order not to restrict the relative energy acceptance Δ and with it the Touschek lifetime, and to not scrape the injected beam.

The maximum horizontal deviation of a particle has several components:

- Betatron oscillation of particles that underwent Touschek scattering somewhere else in the ring with the maximum amplitude given by maximum value of the lattice invariant $\mathcal{H}$ and the required energy acceptance. Translating to the locations of the septum (S) and to where the maximum aperture is required (A) we get $x_{sh} = \sqrt{\mathcal{H}\beta_s}\Delta$, resp. $x_{ah}(s) = \sqrt{\mathcal{H}\beta(s)}\Delta$.
- The dispersive orbit offset is given by $x_{s\eta} = \eta_s\Delta$, resp. $x_{a\eta}(s) = \eta(s)\Delta$
- Since the Touschek scattered particle may also come with an initial betatron amplitude, we have to include the N-sigma beam stay clear: $x_{s\beta} = N\sqrt{\epsilon_s\beta_s}$, resp. $x_{a\beta}(s) = N\sqrt{\epsilon_s\beta(s)}$. Note that N could be chosen larger than N_s, the stored beam stay clear during injection, where one might accept some scratching. The dispersive contribution to beam size is already covered by the Touschek particle amplitude.
- Considering the full capture of the injected beam, we need at least $x_{is} = \sqrt{A_x\beta_s} + N_i\sigma_i\eta_s$, resp. $x_{ia}(s) = \sqrt{A_x\beta(s)} + N_i\sigma_i\eta(s)$.
- Finally, some orbit distortion x_{co} may be added based on simulations of possible orbit excursion *between* BPMs and assumptions on BPM imperfections.

For the Touschek scattered particles, the maximum excursion at the septum thus is given by $x_{\text{Touschek},s} = x_{sh} + x_{s\eta} + x_{s\beta} + x_{co}$.

The maximum excursion somewhere else in the ring requires finding the maximum $x_{\text{Touschek},a} = \max_s\{x_{ah}(s) + x_{a\eta}(s) + x_{a\beta}(s)\} + x_{co}$. This is simplified by assuming that the maximum values of beta and dispersion occur at the same location, which is more or less the case for most light source lattices

For capture of the injected beam, we get
$x_{\text{Injection},s} = x_{is} + x_{co}$, resp. $x_{\text{Injection},a} = \max_s\{x_{ia}(s)\} + x_{co}$.

Finally, minimum septum distance x_s and minimum vacuum chamber half width x_a (assumed constant around the ring) are given by $x = \max\{x_{\text{Touschek}}; x_{\text{Injection}}\}$.

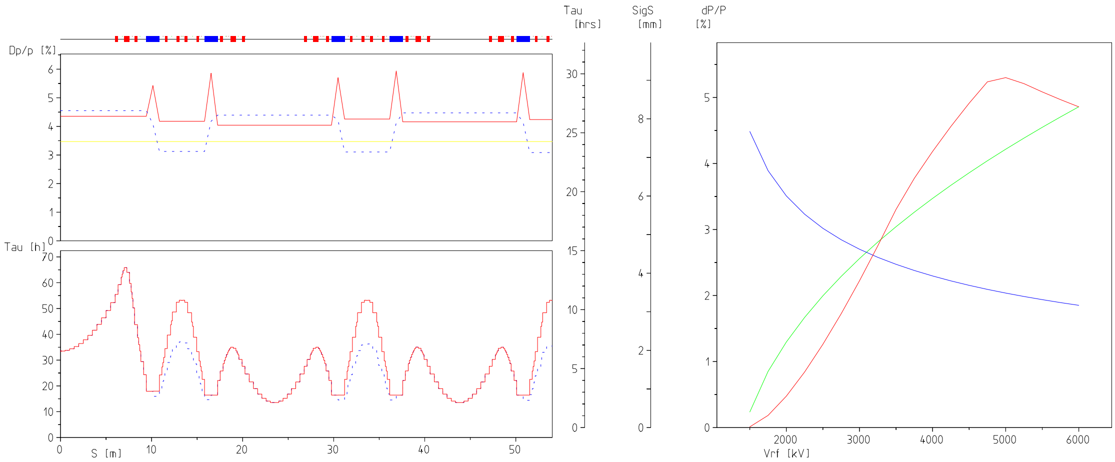

Figure 5: Touschek lifetime calculations for SSRF: right figure shows local ($\pm$) momentum acceptance and lifetime, left figure shows lifetime (red) and energy acceptance (green) vs. RF voltage

Note, that these considerations based on the ideal linear model have to be confirmed by tracking simulations including the non-linear betatron motion as well as the chromatic variation of betafunctions.

The vertical acceptance is determined by the length L and gap g of the insertion device giving the lowest number according to $A_y = (g/2)^2/L$, assuming optimum beta-matching for largest acceptance. This translates to maximum vertical excursion of $y = \sqrt{A_y \beta_{y,\max}}$ at the location of maximum vertical beta. Something for the orbit should be added too.

Table 1 gives an example for the SSRF-5 lattice, resulting in 15.5 mm septum distance and a beam pipe that could be as small as 50 mm wide by 15 mm high (inner dimensions), provided there are no objections concerning vacuum system and resistive wall impedances.

A Touschek lifetime 4D-calculation based on the SSRF-5 lattice with apertures and septum distance as calculated is shown in figure 5: The local momentum acceptance (right figure) is larger than 3.5% everywhere (except at a few locations due to nonlinear distortions of the betatron oscillation) and physically limited. This becomes clear when looking at lifetime as a function of RF voltage: Saturation above the proposed RF voltage of 4 MV indicates, that the lattice acceptance still does not dominate the energy acceptance. The effective lattice energy acceptance derived from Touschek averages amounts to 3.8 %, larger than the 3.5 % as determined by the RF. With 1 mA per bunch and assuming 1 % emittance coupling, a Touschek lifetime of 24 hours results. Of course, this result requires confirmation from 6D-calculations including lattice imperfections.

Table 1: The SSRF-5 optics as an example for calculation of injection parameters and physical acceptances. `beta_i` is the result of a numerical solution of eq.11.

	ssrf 5 aperture requirements	Input		
D	Thickness of the septum sheet	3	mm	
N_s	Number of sigmas for stored beam when passing by septum	7		
beta_s	Betafunction of stored beam at septum	10.2	m	
epsilon_s	Emittance of stored beam	4	nm rad	
N_i	Number of injected beam when passing by septum	3		
beta_i	Optimum betafunction of injected beam at septum	4.25	m	<--
epsilon_i	Emittance of injected beam	110	nm rad	
sigma_i	Injected beam energy spread	0.0010		
A_x	Acceptance required for capture on momentum	8.61	mm mrad	
delta^acc	Required energy acceptance	3.5	%	
beta_max	Maximum betafunction somewhere in the machine	26	m	
	... dispersion at this location:	0.2	m	
eta_max	Maximum dispersion somewhere in the machine	0.31	m	
	... betafunction at this location:	17.5	m	
H_max	Maximum value of the H-function in the machine	0.005660	m rad	
eta_s	Dispersion at the location of the septum	0.15	m	
	rms relative energy spread of the stored beam	0.00100		
N	Number of sigmas for stored beam everywhere	7		
	offset of dispersive orbit	5.18	mm	
	betatron oscillation of maximum Touschek particles	8.41	mm	
	stored beam clearance	1.41	mm	
	allowed tolerance for orbit deviation	0.5	mm	
	Minimum distance for septum to maintain E-acceptance	15.5	mm	
	Minimum distance of septum for capture of injected beam	9.81	mm	
	Minimum distance of septum	15.5	mm	
	Required bump height	14.09	mm	
	Location of maximum beta			
	offset of dispersive orbit	7	mm	
	betatron oscillation of maximum Touschek particles	13.43	mm	
	stored beam clearance	2.26	mm	
	Location of maximum eta			
	offset of dispersive orbit	10.99	mm	
	betatron oscillation of maximum Touschek particles	11.02	mm	
	stored beam clearance	1.85	mm	
	Capture of injected beam			
	Excursion at max beta location	14.96	mm	
	Inj beam energy spread capture	0.6	mm	
	allowed tolerance for orbit deviation	1	mm	
	Horizontal aperture from max beta location	23.68	mm	
	Horizontal aperture from max eta location	24.86	mm	
	Horizontal aperture from injection capture	16.56	mm	
	Horizontal aperture requirement (half width)	24.86	mm	
	Acceptance limiting ID: full length	4.5	m	
	Acceptance limiting ID: full gap	7	mm	
A_y	Vertical acceptance (assuming optimum beta)	2.72	mm mrad	
beta_y_max	Maximum vertical beta somewhere	16	m	
	allowed tolerance for orbit deviation	0.5	mm	
	Vertical aperture requirement (half height)	7.1	mm	