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SLS dynamic acceptance degradation due to magnet multipole errors

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Introduction

The SLS storage ring as a high brightness light source is very sensitive to all kinds of distortions like alignment errors and higher order multiple errors. The influence of alignment errors was described in the SLS design handbook [3], the influence of mini-gap insertion devices (multipole errors and lattice symmetry breaking) is described in [4]. In this note we investigate the influence from higher order multipole errors in the dipoles and quadrupoles, based on the engineering study carried out by Budker institute, Novosibirsk [2].

SLS Lattice

The lattice has two operation modes, the so-called D0 mode with dispersion free straight sections and an emittance of 3.7 nm at 2.1 GeV, and the D1-mode with small dispersion in the straight sections and an effective emittance of 3.1 nm at 2.1 GeV [3]. During the design of SLS much effort was spent on dynamic acceptance optimisation in order to provide long Touschek lifetime, efficient injection and convenient machine operation. Dynamic acceptances much larger than the physical acceptances as given from the finite beampipe width and height are looked upon as good starting point, such that subsequent degradation due to various erros arrives at dynamic acceptances at least as large as the physical ones.

TRACY-2

TRACY-2 is a Pascal-S beam dynamics programming environment [1]. Dynamic acceptances were taken as measures of performance of the lattice with multipole errors. Dynamic acceptances are calculated by include-files to TRACY-2 in the following way: A binary search is done for dynamic horizontal aperture (with small but nonzero value of the vertical coordinate in order to excite coupling resonances) and vice versa for the vertical. Than a 100 turn tracking is performed and a phase space ellipse is fitted to the Poincare plot at trackpoint. The area of the ellipse is taken as dynamic acceptance. Sometimes motion gets trapped in a resonance „island“ and the ellipse fit returns the island area as acceptance. In this case the tracking is repeated with another betatron phase of the test particle.

Definition of multipole strength

TRACY-2:

$(B_y + iB_x) = (B\rho)\sum_n (ia_n + b_n)(x+iy)^{n-1}$ n = 1,2,3... (dipole, quadrupole, sextupole)

with a_n the skew and b_n the regular multipole strength.

The poletipe field (at radius R) of a regular 2n-pole thus is given by $B_y\big|_R = (B\rho)\, b_n\, R^{n-1}$.

If we consider pure 1-dimensional calculations we find for the fields from regular multipoles:

- pure horizontal (y = 0): $B_y = (B\rho)\sum_n b_n x^{n-1}$ = real for any n,
- pure vertical (x = 0): $B_x = -(B\rho)\sum_n b_n i^n y^{n-1}$ = real for even n only.

Thus pure vertical acceptance, calculated for x ≈ 0, will be affected onyl the 2n-poles, where n is even, i.e. quaadrupole, octupole, dodekapole etc., whereas horizontal acceptance will be affected by any multipole.

Novosibirsk magnet study [2]:

A different definiton of multipole coefficients is used with $n_{Novosibirsk} = n_{TRACY} - 1$. We prefer to use the multipole field components for a particular radius as also given in absolute values for the dipoles at 2.1 and 2.4 GeV (p.13, 14) or in relative values for the quadrupoles (p.25 in [2]).

Relative multipole strength:

For input into TRACY we use the ratio of the parasitic multipole strength to the design multipole in order to scale it with excitation of the magnet:

$$\frac{b_n}{b_N} = \left.\frac{B_y^{2n-pole}}{B_y^{2N-pole}}\right|_R \frac{R^{N-1}}{R^{n-1}} \rightarrow \text{ Dipole: } \frac{b_n}{b_1} = \left.\frac{B_y^{2n-pole}}{B_y^{Dipole}}\right|_R \frac{1}{R^{n-1}} \text{ , Quadrupole: } \frac{b_n}{b_2} = \left.\frac{B_y^{2n-pole}}{B_y^{Quadrupole}}\right|_R \frac{1}{R^{n-2}}$$

Magnets' multipole contents according to the Novosibirsk study

Dipole (2.4 GeV, 1.4 T)

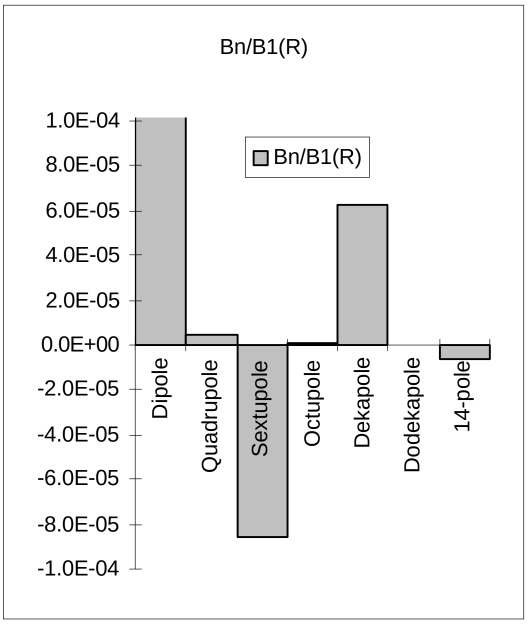
R = 20 mm

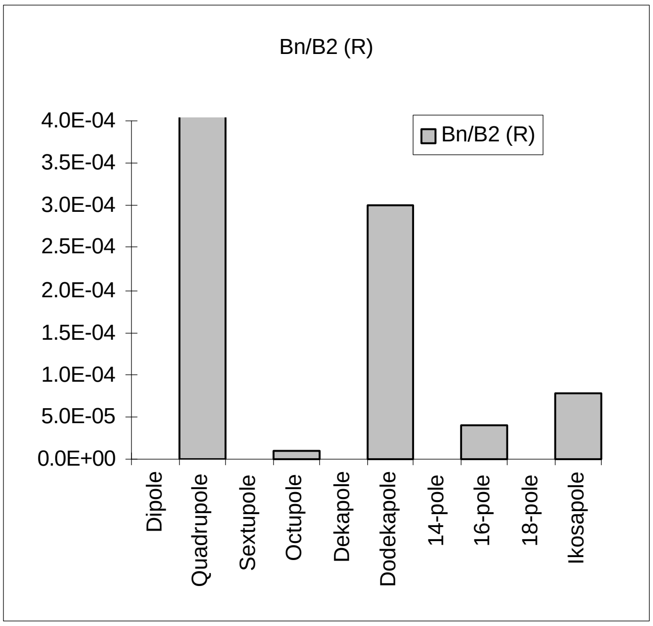
Multipole	**n**	**B_n (R) [T]**	**B_n/B_1 (R)**	**b_n/b_1 [m^{1-n}]**
Dipole	1	1.39797	1	1
Quadrupole	2	5.77E-06	4.13E-06	2.06E-04
Sextupole	3	-1.20E-04	-8.57E-05	-2.14E-01
Octupole	4	1.02E-06	7.30E-07	9.12E-02
Dekapole	5	8.64E-05	6.18E-05	3.86E+02
Dodekapole	6	0	0.00E+00	0.00E+00
14-pole	7	-8.25E-06	-5.90E-06	-9.22E+04

Quadrupole (max. gradient)

R = 30 mm

Multipole	**n**	**B_n/B_2 (R)**	**b_n/b_2 [m^{2-n}]**
Dipole	1	0	0
Quadrupole	2	1	1
Sextupole	3	0	0
Octupole	4	1.00E-05	1.11E-02
Dekapole	5	0	0
Dodekapole	6	3.00E-04	3.70E+02
14-pole	7	0	0
16-pole	8	4.00E-05	5.49E+04
18-pole	9	0	0
Ikosapole	10	8.00E-05	1.22E+08





No multipole decomposition of sextupoles is given in the study, only the general statement that the multipole content is below 10^{-3}. Thus multipoles in sextupoles as well as multipoles in the correctors were not included in these calculations.

Calculations for the lattice in D0 mode

Dynamic acceptances with misalignments and multipole errors

Figures 1 and 2 show the degradation of dynamic acceptances with increasing magnitude of the multipole errors, where 100% correspond to the values given in the tables above. Errors were only applied to dipoles and quadrupoles. Shown in the figure are acceptances for the perfectly aligned machine and 10 seeds and their average for Gaussian distributed (cut at 2 sigma) misalignments of r.m.s. 200 micron for the girder joints, 10 micron for the joint play and 30 micron for the elements relative to the girders. Figure 1 shows the results for TRACY-2 running in transfer matrix mode (with a single multipole kick in the magnet centres) and figure 2 shows the same when TRACY-2 uses the 4^{th} order symplectic integrator for tracking through thick multipoles. Here the results are almost identical, justifying the use of the less precise but much faster matrix method, however later for tracking off-momentum there were some inconsistencies with the matrix mode, and the symplectic integrator was always used.
We see here serious degradation of horizontal acceptance with increasing multipole strength.

Momentum dependant acceptances

Figures 3 and 4 show horizontal and vertical dynamic acceptance as a function of momentum deviation for the ideal lattice, with multipole errors as given in the table above, and with multipole errors and misalignments as before (6 seeds and average). Also shown is the dynamic acceptance of the ideal lattice when including the physical aperture limitations from the beampipe of 65 mm full width and 35 mm full height. This corresponds roughly to the physical acceptance of the linear machine, since for the ideal lattice the dynamic acceptance was shown to be larger than the physical one [3].

The vertical acceptance appears to be rather robust and stays well beyond the physical limitations from the beampipe, whereas the horizontal acceptance is decreased substantially but stays almost as large as the physical acceptance.

Note: Physical acceptance decreases with (positive or negative) momentum deviation due to momentum dependant beating of betafunctions restricting the maximum particle amplitudes. In addition in the horizontal it also decreases due to dispersion shifting the closed orbit towards the beampipe wall with increasing momentum deviation.

Momentum dependant tune

Figure 5 and 6 show the lattice tunes (i.e. tunes of closed orbit) as a function of momentum deviation with and without multipole errors: Obviously the tune is determined by quadrupoles and sextupoles and the higher order multipoles have little influence. Thus the decrease of acceptance is rather due to multipoles driving higher order resonances than detuning of the lattice towards a resonance through the additional multipole errors.

Figure 7 shows the working point as a function of momentum deviation. We see the beam crossing several systematic resonances. Of course most serious is the half integer $2Q_x = 41$, however it interferes with the beam only for momentum deviations beyond 5.5 % dp/p. Other resonances closer to the beam core are of 4^{th} and higher order, in the ideal lattice only weakly driven by sextupoles in 2^{nd} and higher order. However additional higher multipoles from magnet errors drive them directly and thus probably cause the dynamic acceptance degradation.

The isolated effect from dodekapole moments in the quadrupoles was investigated with first suspicion that they were most responsible for spoiling horizontal acceptance (which turned out not to be the full truth, see next section): Figures 8 and 9 show horizontal and vertical tune shifts for the closed orbit with all multipoles, with all except dodekapoles and with dodekapoles only. The dodekapole contributes most to the tuneshift for large momentum deviations, i.e. where the beam »sees« the dodekapole ($B_y \propto x^5$!) due to large dispersive amplitudes, however the contribution is small anyway (see figures 5 and 6).

Who is the »bad guy«?

In order to distinguish the influences from different multipoles the first step was to repeat the acceptance calculations with either only dipole or only quadrupole multipole errors set. The results are shown in Figure 10 for the horizontal acceptance only since the vertical acceptance was robust even with all errors (see figure 4). Obviously the dynamic acceptance degradation is completely dominated by the multipole erros in quadrupoles, whereas the dipole virtually has no effect.

As to be seen from the table above the quadrupoles contain substantial dodekapoles and ikosapoles, smaller hexadekapoles and very small octupoles. Figure 11 shows the horizontal dynamic acceptance when excluding one of the multipole errors in the quadrupole. The octupole was tested too, but had no effect at all and therefore is not shown in the figure. It can be seen from figure 11 that exclusion of only one multipole does not recover the acceptance, i.e. there is more than one multipole responsible for degradation.

Finally for the calculation shown in figure 12 only one multipole was switched on: The hexadekapole is relatively harmless as expected, because it is small. The ikosapole turns out to affect the acceptance most, even more than the dodekapole. Since the ikosapole was the highest pole given in [2] we do not know how even higher multipole might affect the acceptance.

The limited scope for multipoles

For an ikosapole the field rises $B_y \propto x^9$, i.e. up to the specified good field region it is very small and beyond it rises steeply like a »soft wall«.However the increase of higher multipole contributions beyond the good field region reflects nothing but the proximity of the iron poles and thus is more related to *physical* aperture. Basically it makes not much sense to do tracking with higher multipoles outside the quadrupole inscribed radius, because the multipoles are not specified for this region: they are just a *fit* to the measured or simulated magnetic field *inside* the quadrupole bore. Since the higher multipoles tear off only those parts of dynamic acceptance that would be outside the beampipe anyway, the dynamic acceptance degradation we observe is less to worry about than it looks. Figure 13 compares the acceptance results with and without multipole errors including the phyical limitations from the beampipe: Now the degradation due to multipole errors is negligible.

Calculations for the lattice in D1 mode

Figure 14 corresponds to Figure 1 (D0 mode) for the lattice in D1 mode, i.e. with distributed dispersion. This lattice provides larger dynamic acceptance than the D0 mode and slightly smaller longitudinal acceptance due to the vicinity of an integer resonance [3]. The effects from multipole errors are similar to the D0 mode: Significant decrease of horizontal acceptance whereas the vertical acceptance is rather stable to multipoles as well as to alignment errors.

Conclusion

Dynamic acceptances as measures of storage ring performance were calculated with multipole errors in dipoles and quadrupoles according to the Novosibirks engineering study:

- Horizontal acceptance shows degradation by almost a factor of 2 compared to the ideal lattice, down to values that are comparable to the physical acceptance of the beampipe.
- Vertical acceptance is virtually not affected.
- Tune shift due to multipole errors is very small.
- The multipole errors of the dipoles have no effect. The multipole errors of the quadrupoles are completely dominant.
- The ikosapole in the quadrupole has the strongest effect, closely followed by the dodeka-pole. Nothing is known about higher multipoles.
- With alignment and multipole errors the horizontal dynamic acceptance drops beneath the physical acceptance for some values of momentum deviation.
- Influence of multipoles in sextupoles, synchrotron oscillations, tilt errors and location of working point was not yet considered and needs further investigation.
- Both magnet types, dipoles and quadrupoles are acceptable.

References

[1] J. Bengtsson, TRACY-2 user’s guide, unpublished

[2] Engineering study report on the magnetic elements and girders for Swiss Light Source, Budker Institute Novosibirsk, 1997

[3] SLS Design Handbook, chapter 2.2

[4] L. Tosi and A. Streun, SLS dynamic aperture with minigap insertion devices, too be pubslished soon in the SLS-TME-TA series

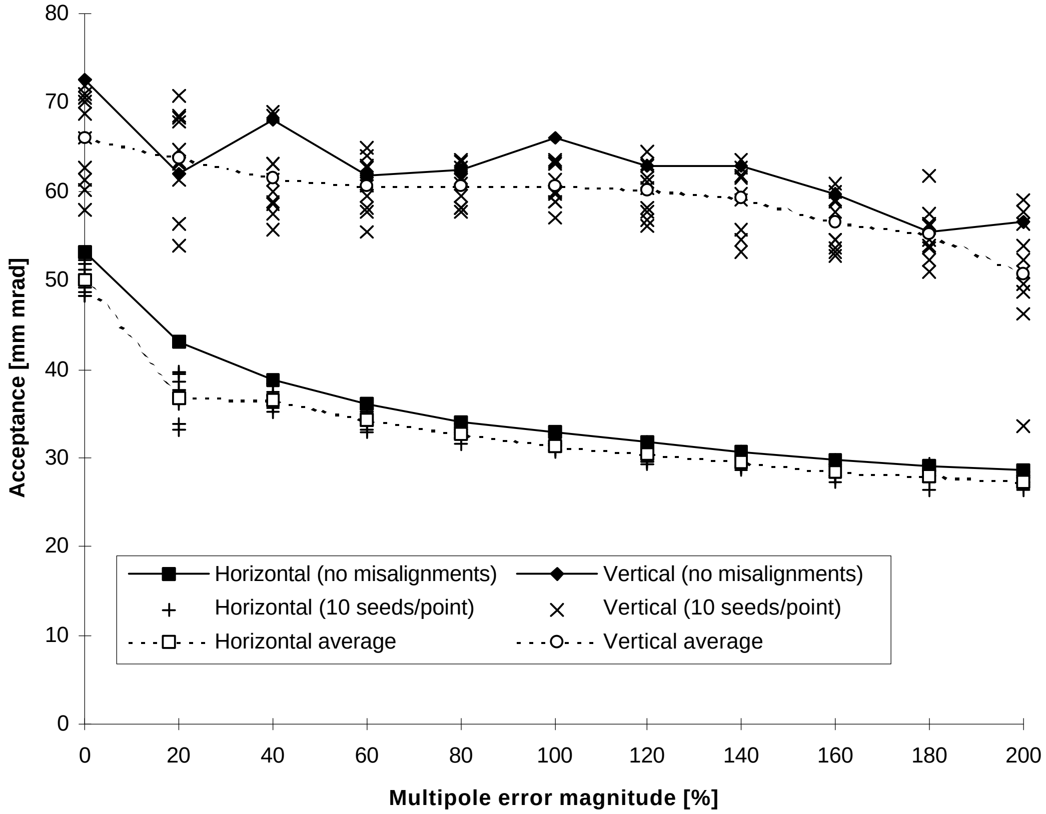


Figure 1: Lattice in D0-mode, TRACY-2 in matrix mode

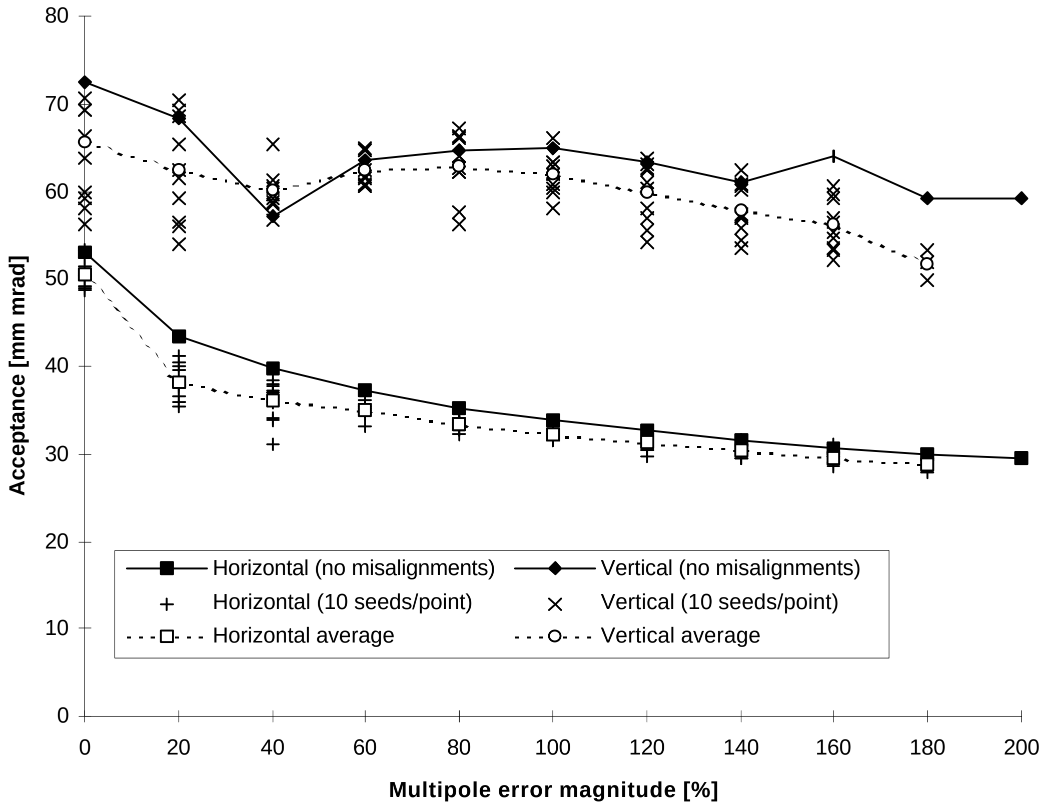


Figure 2:Lattice in D0-mode, TRACY-2 in 4th order symplectic integrator mode

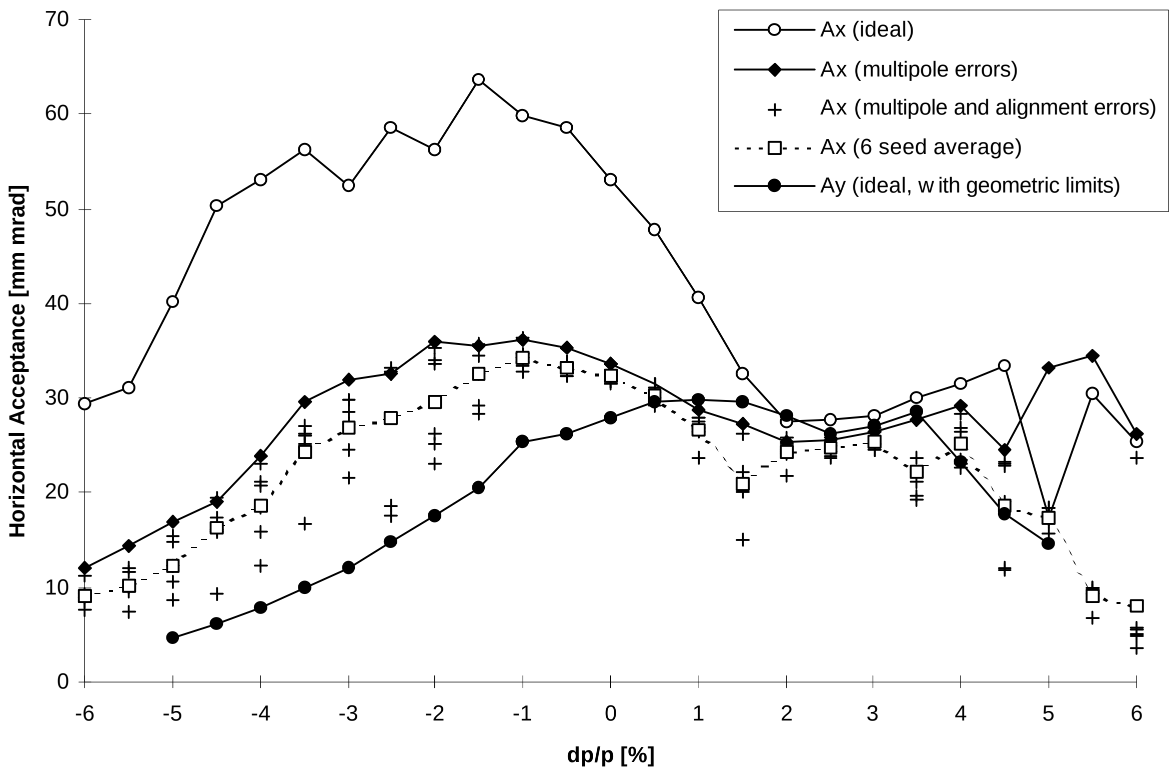


Figure 3: Horizontal acceptance as a function of momentum deviation (D0, SI4)

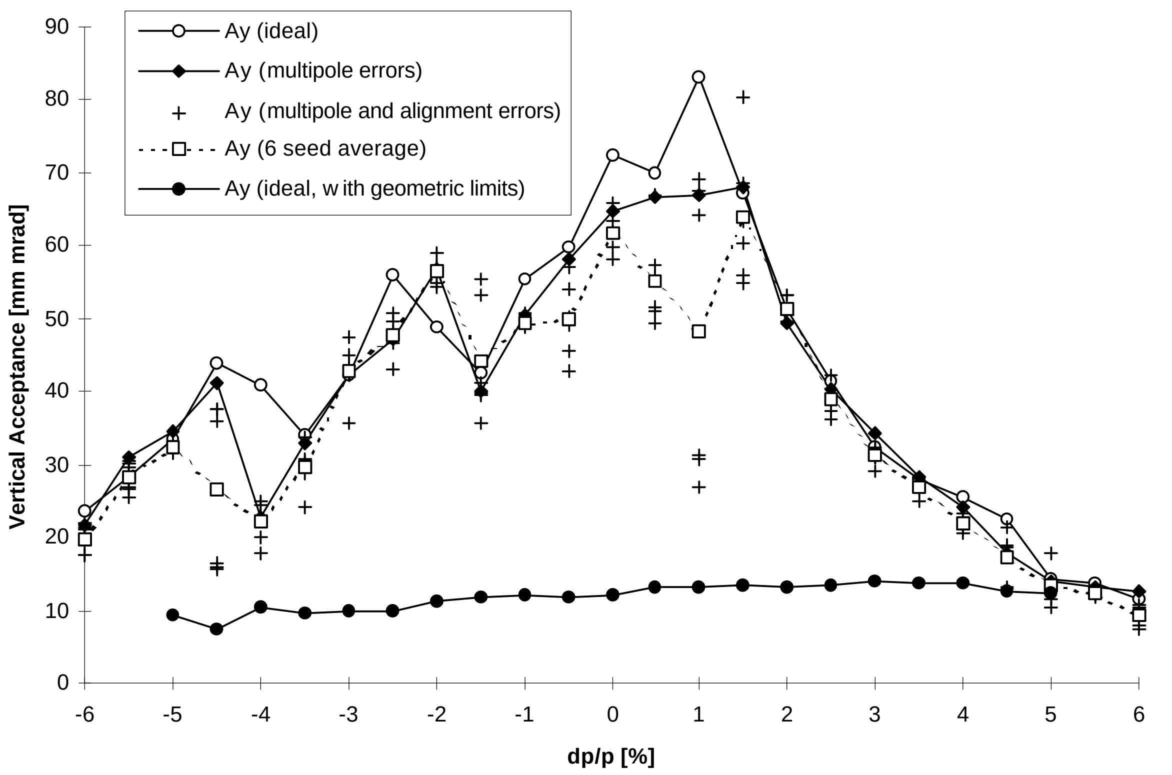


Figure 4: Vertical acceptance as a function of momentum deviation (D0, SI4).

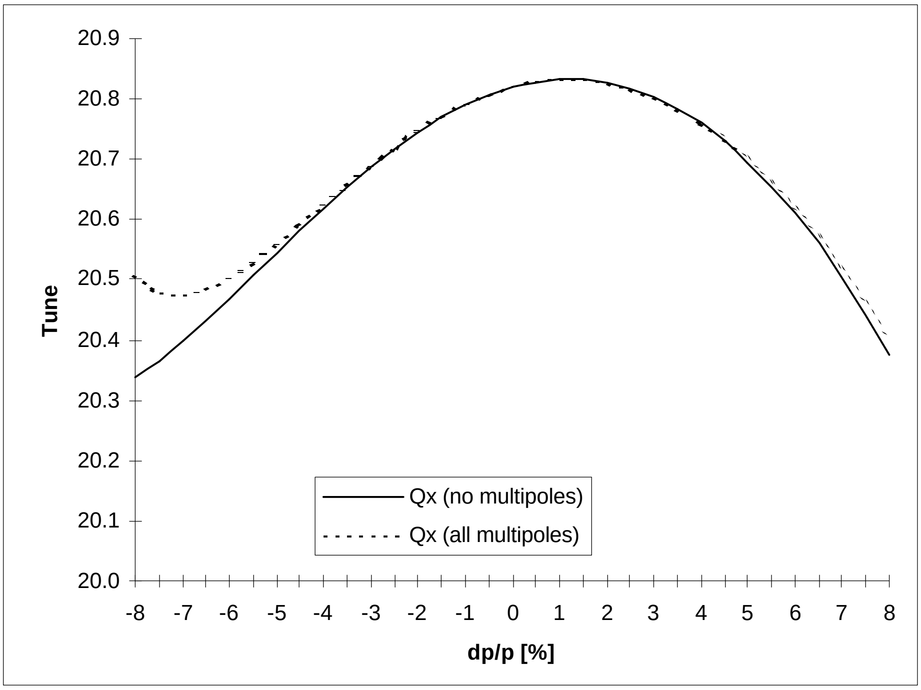


Figure 5:
Horizontal tune as a function of momentum deviation with and without multipole errors.

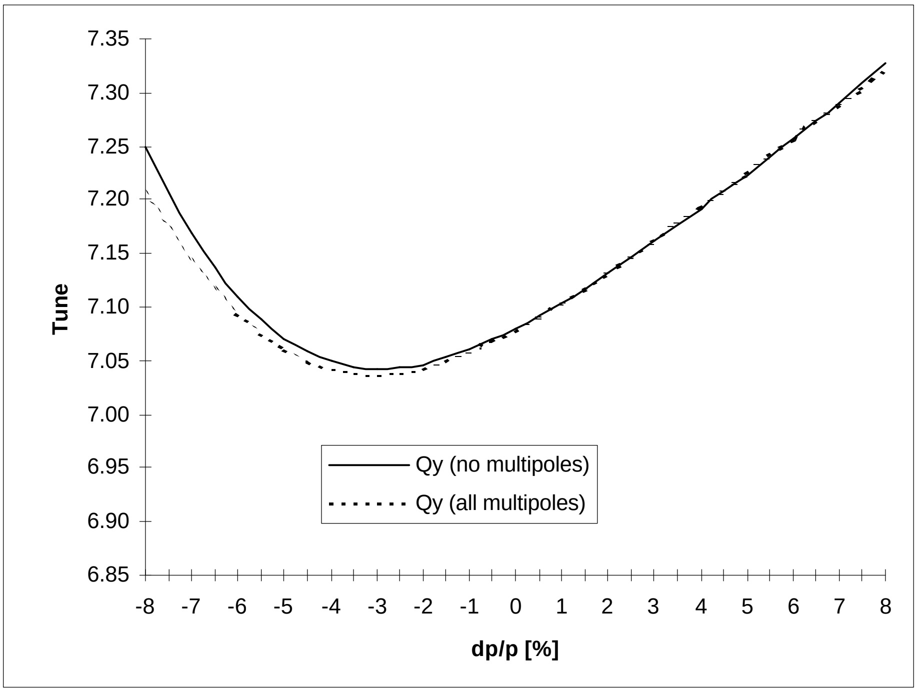


Figure 6:
Vertical tune as a function of momentum deviation with and without multipole errors.

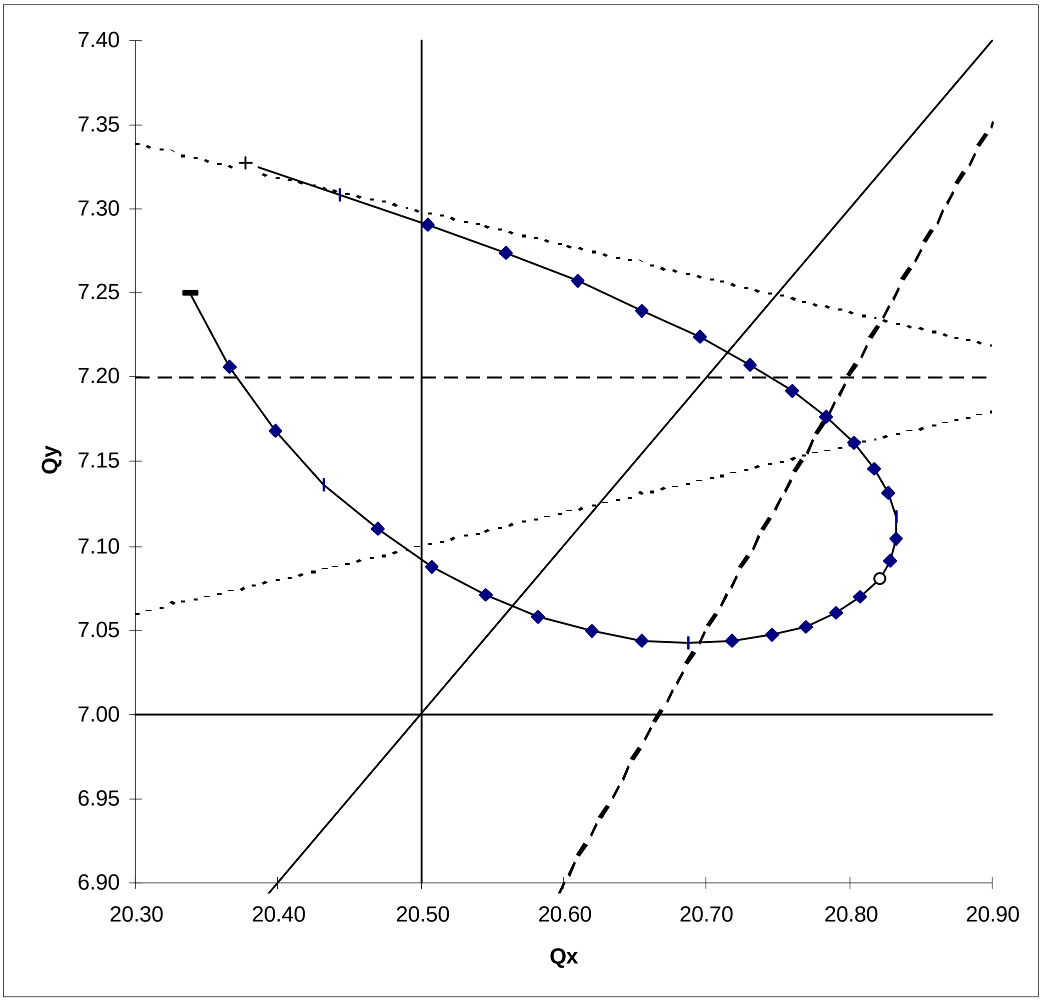


Figure 7: Working point as a function of momentum deviation:

The design working point at 20.82 / 7.08 is marked by o (0% dp/p). The working points for ±8% dp/p are marked by + and – resp., with the points between spaced by 0.5% in dp/p.

Solid lines show systematic resonances up to 4th order:

half integer $2Q_x = 41$ meeting the beam at –5.5 % and +7.0 %,
sextupolar coupling resonance $Q_x - 2Q_y = 6$ at –7.3 % and +6.5 %,
octupolar coupling resonance $2Q_x - 2Q_y = 27$ at ±4.7%.

Dashed lines mark systematic resonances of 5th order:

$3Q_x - 2Q_y = 48$, meeting the beam at –2.9 % and +3.5 %
$5Q_y = 36$, meeting the beam at –7.5 % and + 4.3 %

Dotted lines are 6th order systematic resonances:

$Q_x - 5Q_y = -15$, meeting the beam at –5.7 % and +3%
$Q_x + 5Q_y = 57$, meeting the beam at +7.7 %

Non systematic resonances are not shown since the multipole errors are systematic errors and thus cannot break the lattice symmetry.

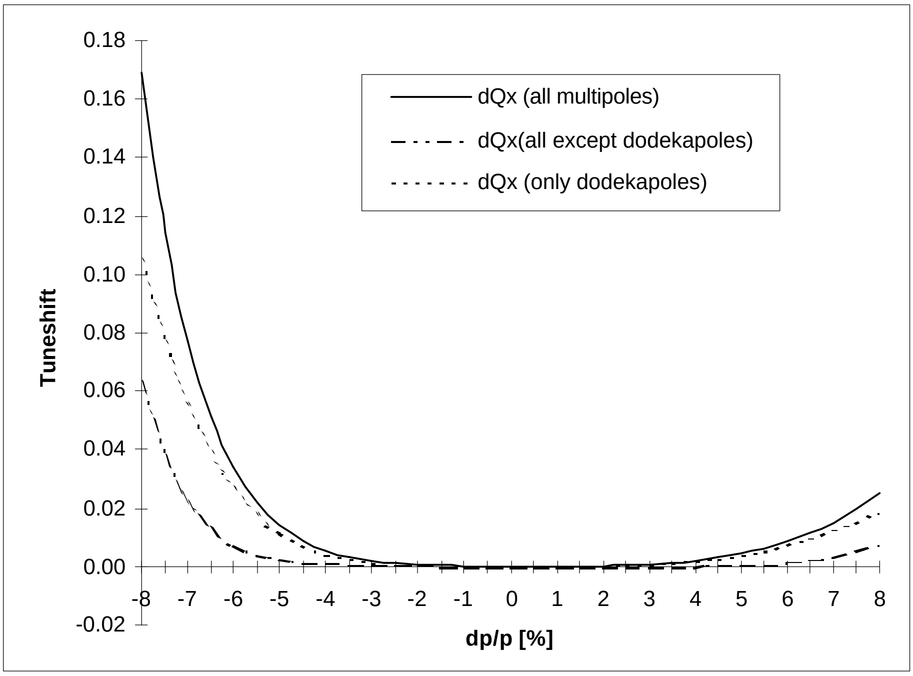


Figure 8: Horizontal Tune shift (= difference to ideal lattice tune, see figure 5) for different multipole error settings: all multipoles, all except dodekapole components in quadrupoles, and only dodekapole components in quadrupoles.

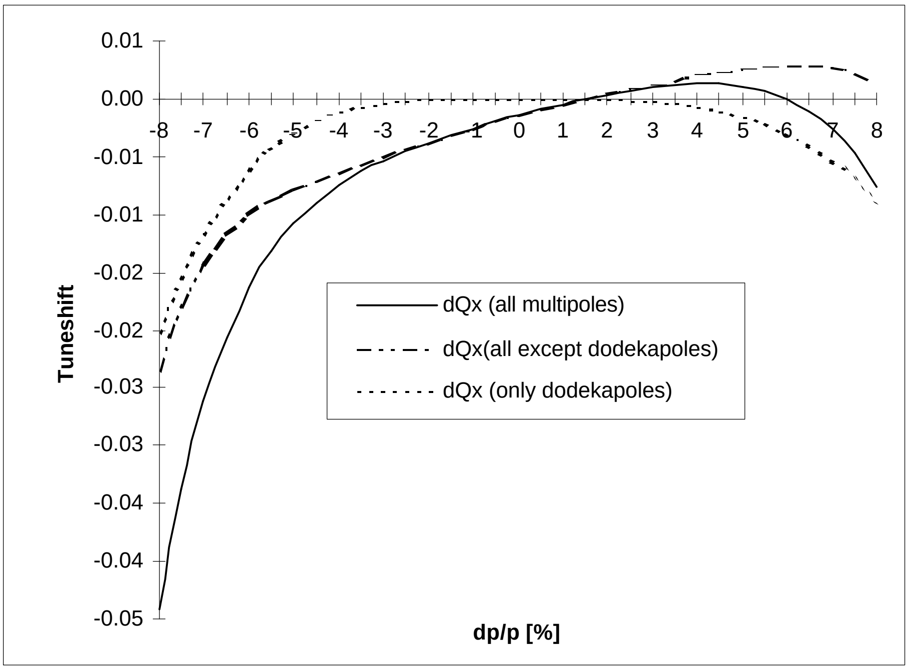


Figure 9: Vertical Tune shift (= difference to ideal lattice tune, see figure 6) for different multipole error settings. Note the very small tune shift for on-momentum (0 % dp/p), caused by the dipoles’ tiny quadrupole component.

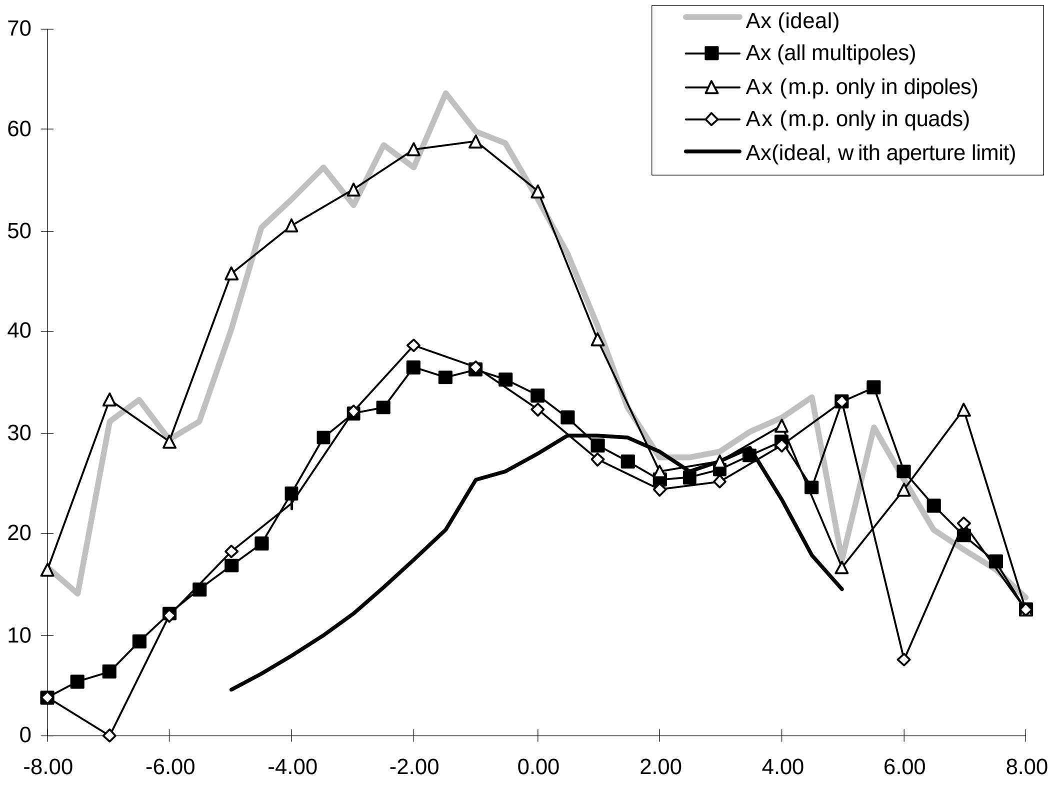


Figure 10: Dynamic horizontal acceptance [mm mrad] vs. momentum deviation dp/p [%] for multipole errors in dipoles, in quadrupoles and in both magnets.

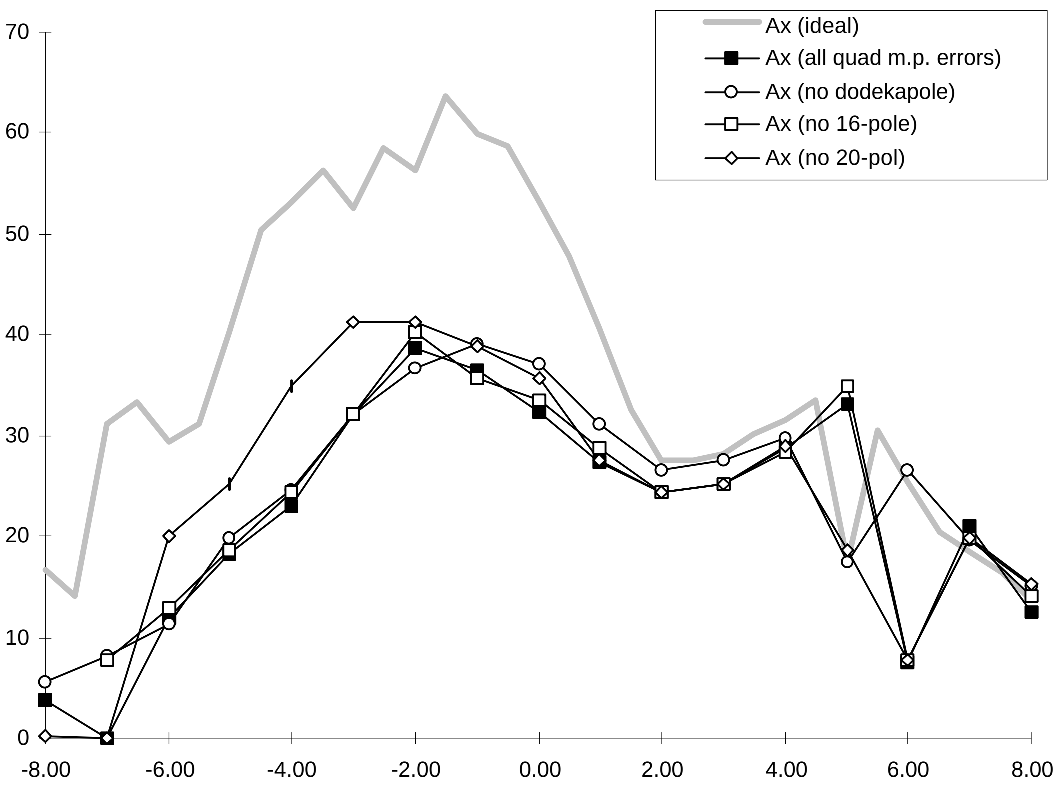


Figure 11: Dynamic horizontal acceptance [mm mrad] vs. momentum deviation dp/p [%] for the ideal lattice, with multipoles in quadrupoles and when excluding one type of multipole error.

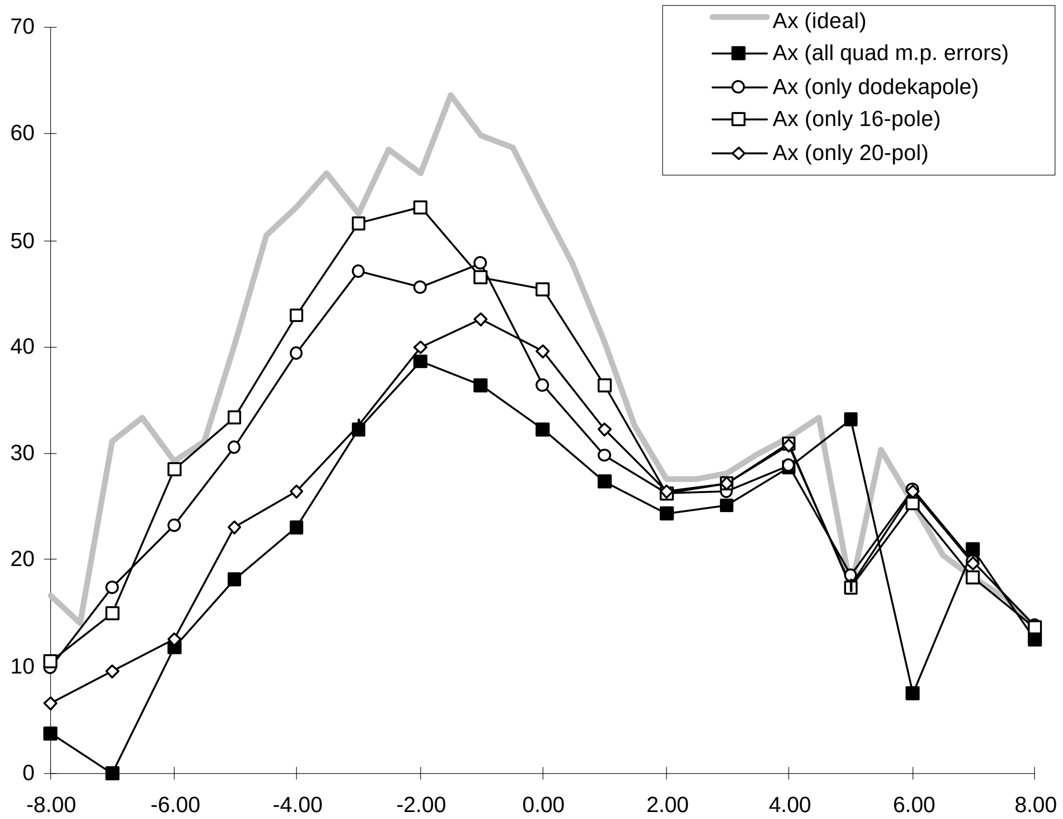


Figure 12: Dynamic horizontal acceptance [mm mrad] vs. momentum deviation dp/p [%] for the ideal lattice, for single and for all quadrupole multipole errors (D0 lattice mode)

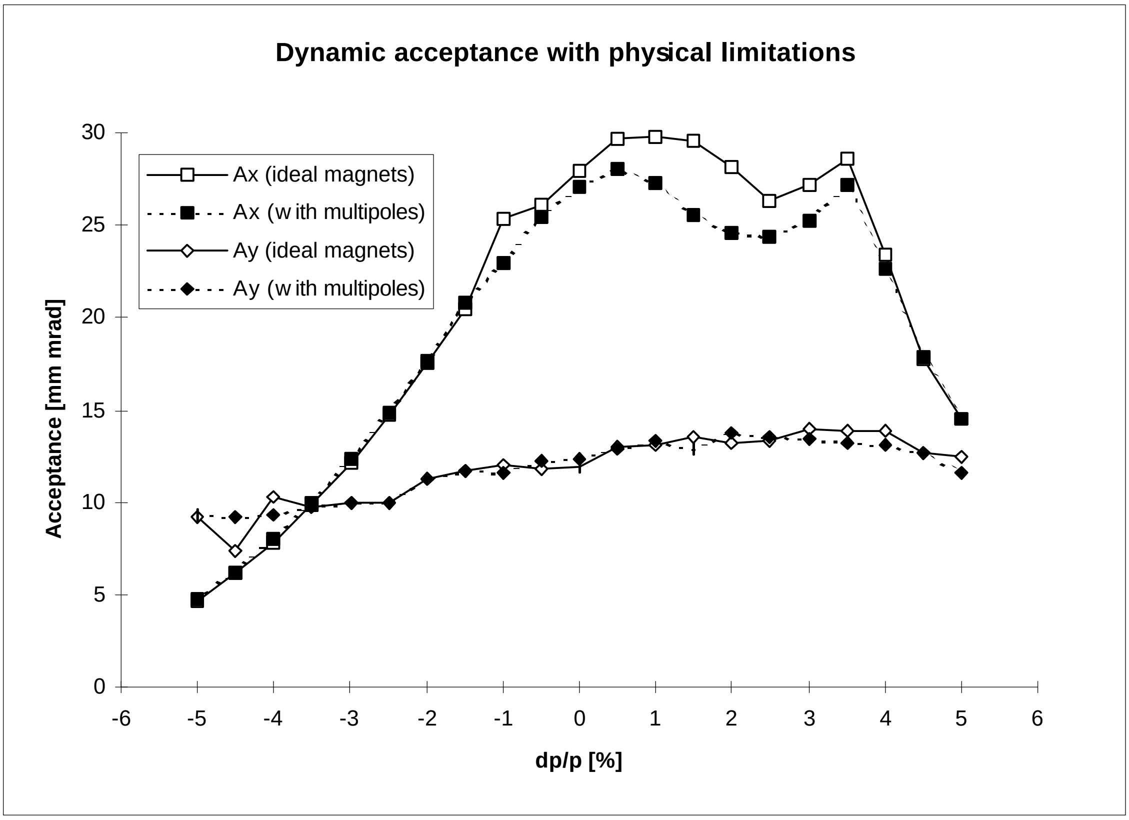


Figure 13: Results for horizontal and dynamic acceptance without and with multipole errors in dipoles and quadrupoles, calculated under physical limitations from a beampipe of 65 mm full width and 35 mm full heigth. (D0 lattice mode)

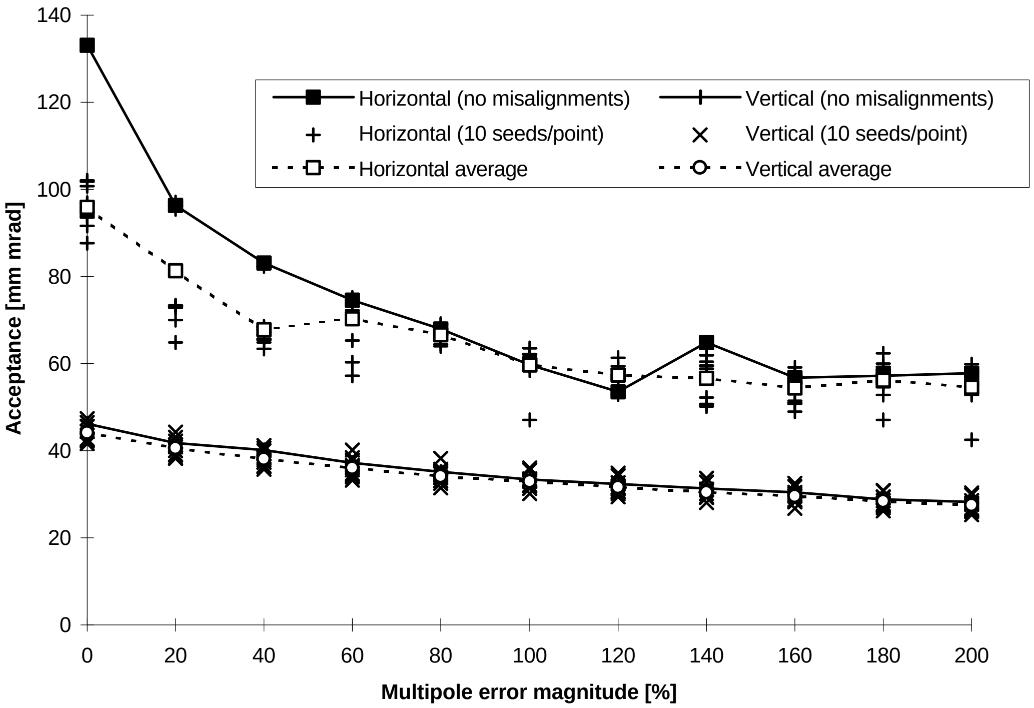


Figure 14: Lattice in D1-mode, TRACY-2 in matrix mode. 100% corresponds to full multipole error setting. Alignment errors were 200 / 10 / 30 micron for girder joints, joint play and elements on girders (r.m.s., <2 sigma).