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Ion trapping in SLS-2	**SLS-2**
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Albin Wrulich	**SLS2-WA80-001-1**
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Annex to SLS-2 Conceptual Design Report

Zusammmenfassung / *Summary*

Most effective ion clearing can be achieved by introducing a gap in the bunch pattern. A gap of 10 empty buckets is sufficient to clean most of the ions at all different optical positions of SLS-2. Fine tuning of gap and/or beam current or emittance ratio can eliminate ion masses still present in the real gas composition. In addition, the nonlinearities of the space charge force would clear ions which are linearly stable and created at positions corresponding to only 10% of one standard deviation.

1. Introduction

Positive ions created by beam gas scattering can be trapped within the bunch by the Coulomb force of the electron bunch resulting in several adverse effects on beam performance: "the local gas pressure is enhanced which reduces the lifetime, the beam size can be blown up by the trapped ion cloud, the tune is changed and a tune spread is generated, nonlinear resonances can be excited, a beam coupling is generated, the γ-Bremsstrahlung is enhanced and a coherent instability can be exited" [1,2,3].

Ion trapping is reduced if the beam size is small, i.e. the space charge forces are strong and if a gap of empty buckets is introduced in the regular bunch pattern. Consequently the problem is particularly harmful for small machines with large emittances. In fact, SLS-1 is only sensitive to ion trapping under certain machine conditions and was suffering from it in the early days, when the vacuum was still not sufficiently conditioned.

For SLS-2 ion trapping should even be less of a problem since the emittance is considerably smaller. Nevertheless it is important to understand how the machine behaves at lower currents and large emittance coupling rations and what gap is actually needed to clear the ions. The results of these studies are presented in the following. All have been performed for the reference lattice of SLS-2 [4] with the following main parameters:

Circumference	C_0=290.4 m
Harmonic numbers	h=484 (f_{RF} = 500 MHz)
Average current	I_{av} = 100 mA, 400 mA
Emittance (for I_B=0)	ε_x = 140 pm-rad
Emittance coupling	κ = 0.1, 0.01

2. Ion stability for a bunch pattern with and without gap

2.1 Uniform bunch pattern

In the linear regime, the circulating bunch structure generates a periodic focusing for the ion in both planes. In analogy to the electron beam circulating in the magnet structure, we can define the deflection of the ion in form of a thin lens approximation [2]:

$$\begin{bmatrix}\Delta x' \\ \Delta y'\end{bmatrix} = -\frac{2N_b r_p}{A}\left(\frac{1}{\sigma_x+\sigma_y}\right)\begin{bmatrix}x/\sigma_x \\ y/\sigma_{xy}\end{bmatrix} \tag{1}$$

With N_b the number of particles per bunch, r_p the classical proton radius, $\sigma_{x,y}$ the standard deviations of the transverse electron beam sizes and A the ion mass in atomic units.

The focusing force becomes stronger if the beam size is smaller, i.e. the emittance is smaller and/or the beta value at the point of the ion position is smaller. Consequently, the effect is usually much larger in the vertical plane, where the emittance is a fraction of the horizontal one.

Moreover is the focusing strength directly proportional to the bunch intensity and inversely proportional to the ion mass, which means larger masses are easier trapped than lower masses.

For the computations in this chapter, the position with the maximum vertical beta function of the SLS-2 optics has been chosen, i.e. β_y = 13.3 m (β_x = 1.6 m). Calculations were performed mainly for the less favorable case of 100 mA with 10 % emittance coupling (case A) and for the nominal case of 400 mA beam current with 1 % emittance coupling (case B).

The transverse transformation matrix of an ion by one revolution of a bunch train with N regularly spaced bunches is given by:

$$M_{rev} = (M_d M_b)^N \quad (2)$$

$$\text{with} \quad M_d = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix}, \quad M_b = \begin{bmatrix} 1 & 0 \\ -\alpha & 1 \end{bmatrix}, \quad \alpha = \frac{N_b r_p}{A}\left(\frac{2}{\sigma_x+\sigma_y}\right)\begin{bmatrix} x/\sigma_x \\ y/\sigma_y \end{bmatrix}$$

The motion of the ion is stable, i.e. the ion is trapped, if the trace of the revolution matrix fulfills the condition:

$$-2 < Tr(M_{rev}) < 2 \quad (3)$$

from which the ion mass at the stability limit can be derived. All ion masses larger than that are trapped. For a uniform bunch train the critical mass is given by:

$$A_{limit} = \frac{N_b r_p}{(\sigma_x + \sigma_y)\sigma_y}\frac{d}{2} \quad (4)$$

In Fig. 2-1 the results of the two cases for a uniform filling are presented. For case A only masses up to A=3 are expelled whereas for case B ion masses up to A=12 become unstable and this even for a uniform bunch filling. The red lines indicate the stability limit defined by relation (3).

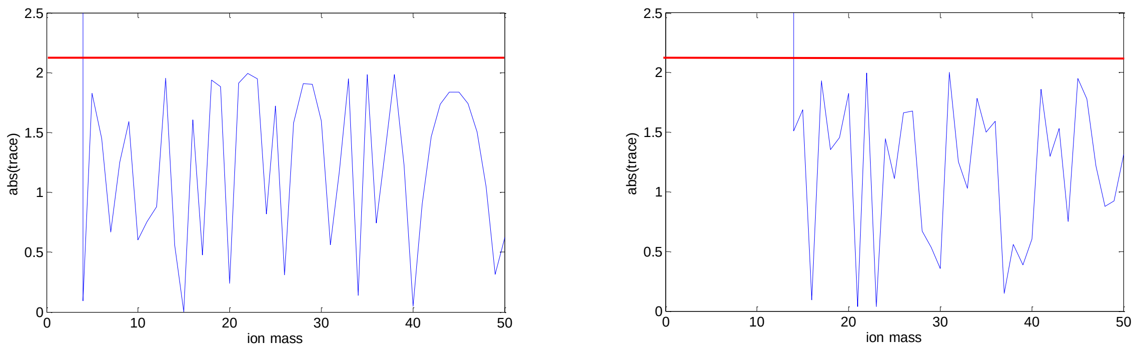

Figure 2-1: Trace of the revolution matrix for a regular bunch filling without gap.

Case A (left): I = 100 mA, κ=0.1; Case B (right): I = 400 mA, κ=0.01.

2. 2 Bunch pattern with gap

If the regular bunch pattern is interrupted by a gap, the "ion-beta functions" are strongly disturbed and can reach high values. Due to the mismatch of the electron phase space and the ion phase space the created ions can escape the linear space charge regime and eventually get lost in the highly nonlinear regime. But even in the linear regime the "ion-optics" might hit a half integer stop band which makes the ion already linearly unstable. It is obvious that the position of the stop band can be shifted by the gap length and the beam intensity which allows the elimination of ion masses present in the residual gas composition (usually: H_2=2, CH_4=16, H_2O=18, CO=28, CO_2=44) by fine tuning of these parameters.

If the regular bunch train of B bunches is interrupted by a gap with length D, the revolution matrix has the form:

$$M = M_D M_b (M_d M_b)^B = \begin{bmatrix} 1-\alpha D & D \\ -\alpha & 1 \end{bmatrix} \begin{bmatrix} 1-\alpha d & d \\ -\alpha & 1 \end{bmatrix}^B \qquad (5)$$

with d the regular bunch distance, h the harmonic number and D=(h-B+1)d.

Fig. 2-2 shows the results for cases A and B for a bunch pattern with a gap of 10 empty buckets (number of filled buckets B=474).

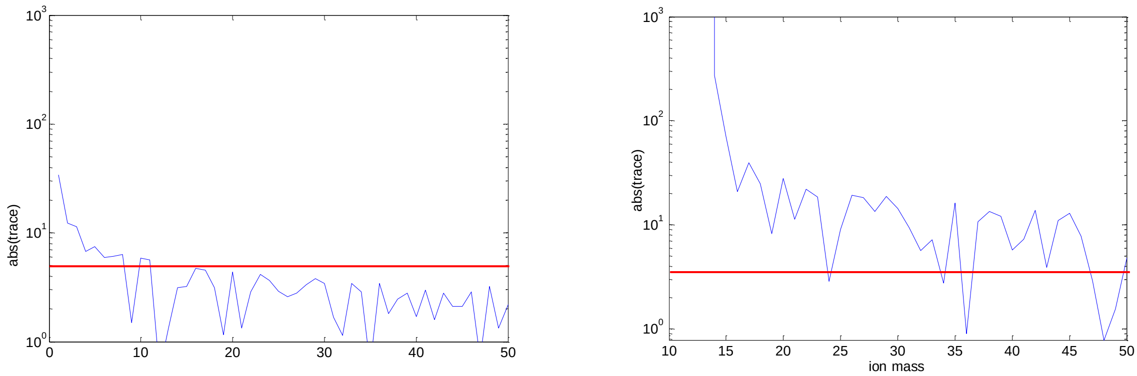

Figure 2-2: Trace of the revolution matrix for a regular bunch fulling with 10 empty buckets.

Case A (left): I = 100 mA, κ=0.1; Case B (right): I = 400 mA, κ=0.01.

3. Ion trapping at different optics positions

In the previous chapter the maximum vertical beta value for SLS-2 has been taken for the evaluation of the ion stability. This is the worst case since the deflections of an ion due to the bunch space charge forces in the horizontal and vertical plane are proportional to the following functions:

$$G_x = \frac{1}{(\sigma_x+\sigma_y)\sigma_x} \qquad G_y = \frac{1}{(\sigma_x+\sigma_y)\sigma_y} \qquad (6)$$

If the emittance ratio is small, the vertical beam size is small and consequently the ion is predominately expelled in the vertical plane (G_y>>G_x). For larger coupling factors on the other hand, as already 10 %, the vertical clearing is not dominating anymore at all lattice positions. Therefore the calculations have been extended to very different beta values of the lattice. Nine representative positions of the nominal SLS-2 optics have been selected which are shown in Fig. 3-1. The corresponding beta values are listed in Tab. 3-1.

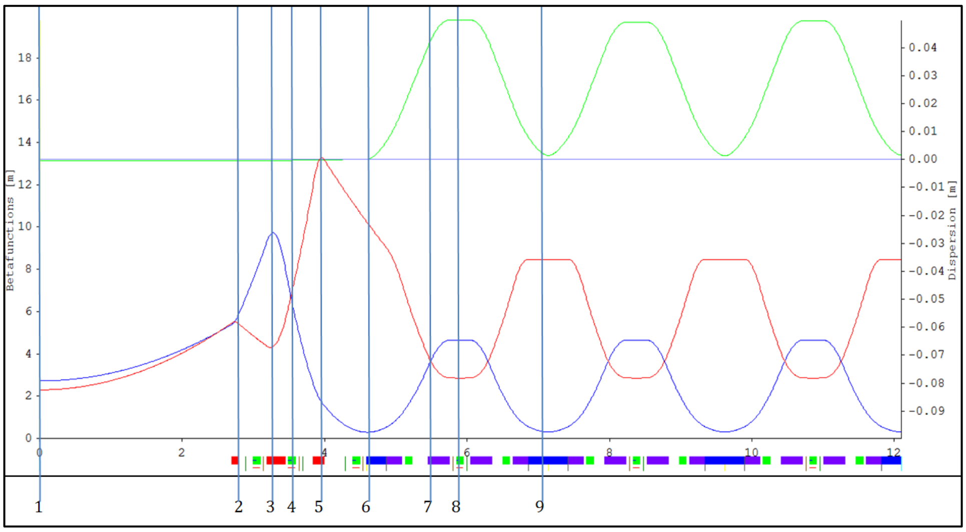

Figure 3-1: Optics positions of SLLLS-2 selected for a detailed analysis

Table 3-1: Beta values at the selected SLS-2 positions

Pos	β_x [m]	β_y [m]
1	2.7	2.3
2	5.9	5.4
3	9.5	4.3
4	6.4	6.9
5	1.6	13.3
6	0.3	10.1
7	3.4	3.9
8	4.6	2.9
9	0.3	8.5

In Tab. 3-2 the clearing results are presented for case B with I=100 mA, 10 % emittance coupling and a gap length of 10 empty buckets. In Tab. 3-3 the results are collected for a uniform filling without gap, but 400 mA and 1 % emittance ratio.

Table 3-2: Stability of ion masses at 9 optical positions for I=100 mA, 10 % emittance coupling and 10 buckets empty

Pos	β_x	β_y	G_x	G_y	Ion masses 1-25																								
	m	m	$m^{-2}/10^8$	$m^{-2}/10^8$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	23	25
1	2.7	2.3	2.0	7.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	5.9	5.4	0.9	3.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	0	0	0	0	0	0	X
3	9.5	4.3	0.6	2.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4	6.4	6.9	0.8	2.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	0	X	0	0	0	0	0	0
5	1.6	13.3	2.3	2.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	0	0	0	0	0	0
6	0.3	10.1	8.4	4.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	3.4	3.9	1.6	4.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	0	0	0	0	0	0	0
8	4.6	2.9	1.2	4.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	0	0	0	0	0	0	0	0
9	0.3	8.5	8.9	5.3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Pos	β_x	β_y	G_x	G_y	Ion masses 26-50																								
	m	m	$m^{-2}/10^8$	$m^{-2}/10^8$	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	2.7	2.3	2.0	7.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	5.9	5.4	0.9	3.1	0	0	0	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X
3	9.5	4.3	0.6	2.9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	0	0	0	X	0	0	X	0	0	0
4	6.4	6.9	0.8	2.6	0	0	0	0	0	0	X	0	X	0	0	0	0	0	X	0	0	X	0	0	0	0	X	0	0
5	1.6	13.3	2.3	2.6	0	0	0	0	0	X	X	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0.3	10.1	8.4	4.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	3.4	3.9	1.6	4.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	4.6	2.9	1.2	4.9	0	0	0	0	0	0	0	0	0	X	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0.3	8.5	8.9	5.3	0	0	X	0	0	0	0	0	0	X	0	0	0	0	X	0	0	X	0	0	0	0	0	0	X

A well-conditioned vacuum would beside hydrogen (A=2) still include CO (A=28) which is is only trapped at one optical position. On the other hand, the longitudinal motion of the ion would move it into an area where it becomes unstable.

Table 3-3: Stability of ion masses at 9 optical positions for I=100 mA, 1 % emittance coupling and 10 buckets empty

Pos	β_x	β_y	G_x	G_y	Ion masses 1-25																								
	m	m	$m^{-2}/10^9$	$m^{-2}/10^9$	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	23	25
1	2.7	2.3	2.4	26.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	5.9	5.4	1.1	11.6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3	9.5	4.3	0.7	10.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	X
4	6.4	6.9	1.0	9.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	X	X	X
5	1.6	13.3	3.5	12.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
6	0.3	10.1	15.1	26.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	3.4	3.9	2.0	17.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
8	4.6	2.9	1.4	18.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
9	0.3	8.5	15.5	29.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Pos	β_x	β_y	G_x	G_y	Ion masses 26-50																								
	m	m	$m^{-2}/10^9$	$m^{-2}/10^9$	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47	48	49	50
1	2.7	2.3	2.4	26.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	5.9	5.4	1.1	11.6	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
3	9.5	4.3	0.7	10.5	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
4	6.4	6.9	1.0	9.7	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
5	1.6	13.3	3.5	12.0	0	0	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
6	0.3	10.1	15.1	26.0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
7	3.4	3.9	2.0	17.7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	X	X	X	X	X	X	X	X	X	X
8	4.6	2.9	1.4	18.1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	X	X	X	X	X	X	X	X	X	X
9	0.3	8.5	15.5	29.2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

4. Ion optics with and without gap

The circulating bunches are manifesting a focusing structure (in both planes) for the ion. The results for a uniform filling and a gap of 10 empty buckets is shown in Fig. 4-1. Even this small gap causes an increase of the beta beat by more than a factor of 20.

In consequence, for the situation with gap, the maximum vertical ion amplitude will depend strongly on the position where it is created in the circulating bunch train. It might reach values far outside the linear regime of the space charge force and the linearly stable ion might therefore be expelled due to the nonlinear character of the space charge forces.

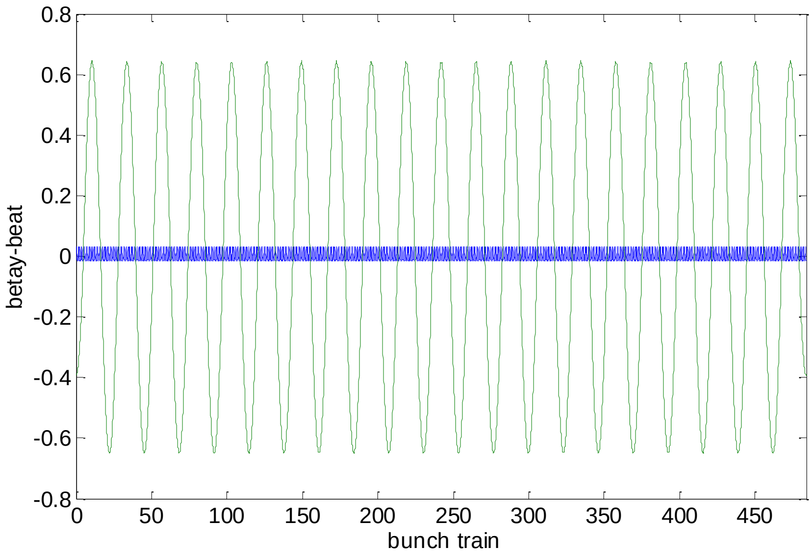

Figure 4-1: Vertical beta functions generated by the focusing structure of the circulating bunches for I=100 mA, 10 % emittance coupling, without gap (blue) and with a gap of 10 empty buckets (green)

The minimum/maximum beta values in figure 4-1 are for instance at bunch positions n_B=242/248 for the vertical plane. Starting point for the numbering is the center of the straight section.

5. Ion trapping with nonlinear space charge force

5.1 Nonliner space charge force

In the most general case the deflection by the space charge force of a Gaussian beam is given by [5]:

$$\begin{bmatrix}\Delta x' \\ \Delta y'\end{bmatrix} = \frac{N_b r_p}{A}\sqrt{\frac{2\pi}{\sigma_x^2-\sigma_y^2}}\begin{bmatrix}-Re(f_{xy}) \\ Im(f_{xy})\end{bmatrix}$$

$$f_{xy} = -i\left[w(z_2) - w(z_1)e^{z_1^2-z_2^2}\right] \qquad w(z) = e^{-z^2}[1-erfz(-iz)] \tag{7}$$

$$z_1 = \left(\frac{x}{\sigma_0}\right)k + i\left(\frac{y}{\sigma_0}\right)\frac{1}{k} \qquad z_2 = \left(\frac{x}{\sigma_0}\right) + i\left(\frac{y}{\sigma_0}\right)$$

$$\sigma_0 = \sqrt{2(\sigma_x^2-\sigma_y^2)} \qquad k = \frac{\sigma_y}{\sigma_x}$$

with erfz as the complex error function.

Considerations with nonlinear space charge forces have only been made for the optical position 8, with the less favorable vertical clearing conditions for I=100 mA and 10% emittance coupling. Fig. 5-1 (left) shows the resulting nonlinear space charge forces for the horizontal and vertical plan. As can be seen, due to the large emittance ratio they are very close together and consequently clearing is not always dominated by the vertical plane.

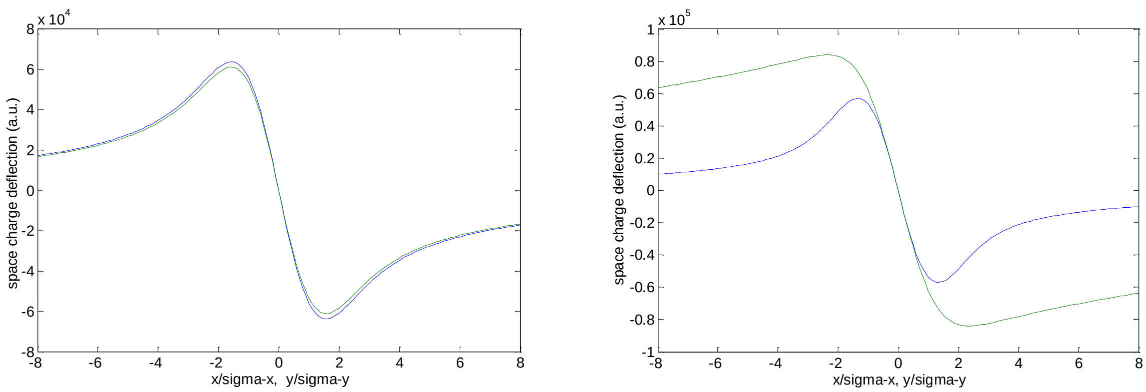

Figure 5-1: Space charge forces for the horizontal (blue) and vertical (green) plane,

I = 100 mA, κ=0.1 (left) / I = 400 mA, κ=0.01(right)

For comparison, Fig. 5-1 (right) shows the same situation with 1 % emittance coupling. Here we have a pronounced difference between the two planes and ion clearing mainly happens in the vertical plane.

As shown in Fig. 5-1, for higher ion displacements from the beam center the attractive Coulomb force is reduced and becomes highly nonlinear. The maximum is reached at $\sqrt{2}\sigma_{x,y}$ which means that the linear approximation is only valid until to ion displacements around $\sigma_{x,y}$, i.e. one standard deviation of the electron beam dimension. A stronger decay with distance for 1% coupling (in terms of sigmas but not in absolute terms) can be observed for the weaker horizontal space charge force.

5.2 Nonlinear ion tracking

Tracking has been performed only for case A (100 mA, 10 % emittance coupling) with a gap length of 10 empty buckets. The ion number 32 has been chosen which lies in the stable regime of Fig. 3-2.

Tracking results are shown in Fig. 5-2. Shown is the area which is populated by the ion over the tracking distance, ie the ion position is drawn for each bunch passage. It turned out, that 10 turns (4740 bunch passages) are sufficient to provide a clear picture of the clearing process.

Fig. 5-2a shows the initial distribution of 30 particles for a 1 sigma distribution. Three different initial amplitudes (Fig 5-2b,c,d) at sigma fractions of 0.001, 0.05 and 0.1 are shown. For the smallest amplitudes the particles only experience the linear force of the space charge and remain stable. The amplitude increase compared to the initial distribution is caused by the mismatch between the lattice optics and the ion optics at the observation point. But already at a sigma fraction of 0.05 (Fig. 5-2c) the ion is pushed into the nonlinear regime and most of initial positions (25 out of 30) are already lost. All ions are cleared for a 0.1 sigma fraction (Fig 5-2d). This agrees also with the observation that the transverse ion distribution is not a replica of the beam distribution. It is composed by a central core and less extended in transverse direction than the electron beam.

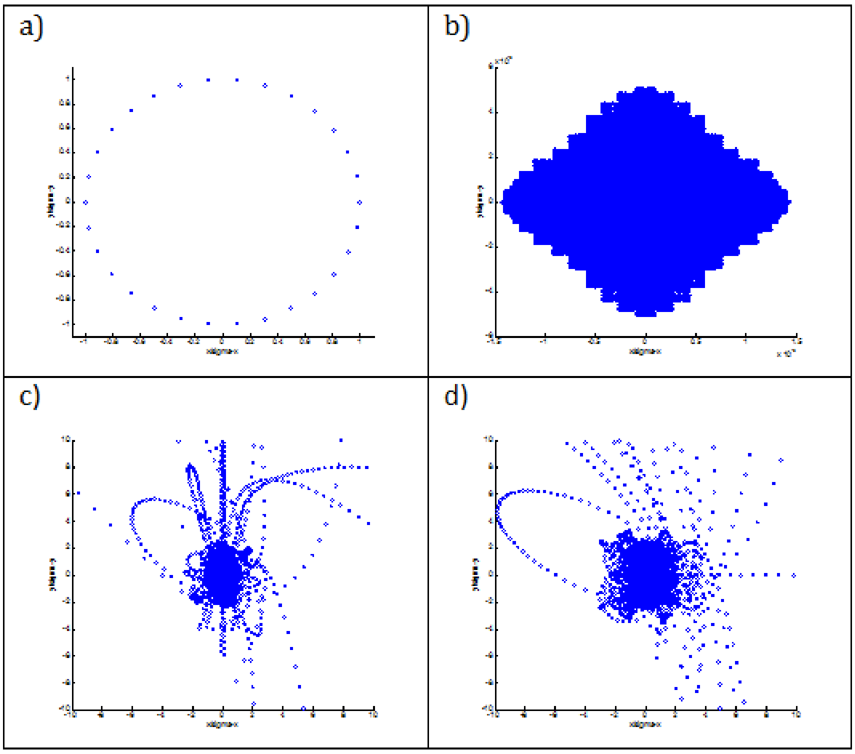

Figure 5-2: Nonlinear ion tracking.

Initial emittance 0.01 sigma (left) and 0.015 sigma (right)

6. Summary and conclusions

For the nominal operational parameters of SLS-2, i.e. 400 mA beam current and 0.01 emittance coupling full ion clearing is guaranteed with even only a small gap in the bunch pattern. But during the initial operations phase only lower beam currents can be stored. Moreover, also during regime operation larger vertical emittances might be preferred in order to combat Touschek scattering. The studies performed here have been mostly limited to the adverse beam conditions during the initial operations phase, i.e 100 mA beam current and 10% emittance coupling.

Most effective ion clearing can be achieved by introducing a gap in the bunch pattern. This generates a strong beta beat of the ion optics. Due to the mismatch between the electron optics and the ion optics the original displacement of an ion which is created by a certain bunch within the bunch pattern can strongly increase and drive the ion in the nonlinear regime of the space charge force where it is lost. Consequently the clearing efficiency for an ion depends by which bunch within the pattern the ion is created. Moreover, also the different optical positions of the SLS-2 lattice show very different trapping behavior. At some positions also relevant clearing in the horizontal plane can occur.

A gap of 10 empty buckets is sufficient to clean most of the ions at all different optical positions of SLS-2. Fine tuning of gap and/or beam current or emittance ratio can eliminate ion masses still present in the real gas composition.

In addition, the nonlinearities of the space charge force would clear ions which are linearly stable and created at positions corresponding to only 10% of one standard deviation.

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