A. Streun, Oct. 2017
Empirical method of particle physics: Accelerators

Exercise 1

A. LHC has 1232 dipole magnets of 8.33 T field strength to store protons of a momentum of 7 TeV/c. What is the (average) length of a magnet?

B. Consider a small ring of 30 m circumference, filled to 60% with dipole magnets of 1.75 T field strength. Calculate kinetic energy and velocity of positrons, protons and fully stripped carbon ions stored in this ring.

Hints:
all formula and data are given on slide 11 and 13.
Using "magnetic rigidity" makes it much easier!

Magnetic rigidity

(also see slide 69)
On a circular path of radius ρ in a magnetic field B, the Lorentz force is equal to centrifugal force:

$$qvB = \frac{mv^2}{\rho}$$

with q, m, v charge, mass and velocity of a particle. This equation is relativistically valid, with $v = \beta c$ and $m = m_o \gamma$.
Introducing the momentum $p = mv$ we get $\boxed{(B\rho) = p/q}$.

The product of field and radius is called **magnetic rigidity** and has the unit T·m. It is useful for calculations like Exercise 1, because we need only momentum and charge.
Charge is a multiple of elementary charge, $q = ne$, and momentum $p = \beta E/c$ is given in practical units of eV/c, not in SI units of kg·m/s:

$$(B\rho)\,[\mathrm{Tm}] = \frac{p\,[\,\mathrm{kg{\cdot}m/s}]}{n\,e\,[\mathrm{A{\cdot}s}]} = \frac{e}{c} \times \frac{p\,[\mathrm{eV/c}]}{n\,e} = \frac{p\,[\mathrm{eV/c}]}{n\,c}$$

With $c \approx 3 \cdot 10^8$ m/s and using GeV, not eV, which is more suitable to accelerators, we finally have a practical formula:

$$(B\rho) \approx \frac{10}{3}\,\frac{p\,[\mathrm{GeV/c}]}{n}.$$

Exercise 2

A linear accelerator of length $L = 1$ km and an average accelerating gradient of $G = 10$ MV/m is used to accelerate muons, which have mass $m_o c^2 = 105.6$ MeV, charge $q = e$ and a mean life time $\tau = 2.2$ μs. Assume that the muons start at rest.

Which percentage of the muons arrives at the end of the linac (at $z = L$)?

Hints:
Decay is described by $N = N_o e^{-t/\tau}$ and happens in the *moving* system of the accelerating particle.
Don't care about the linac structure, cell lengths are assumed to be adjusted to give a constant accelerating

gradient.
You need the relativistic formulae from slide 11, the Lorentz force from slide 26 and the infinitesimal Lorentz transformation $c\,dt = \beta\gamma dz' + \gamma c\,dt'$.

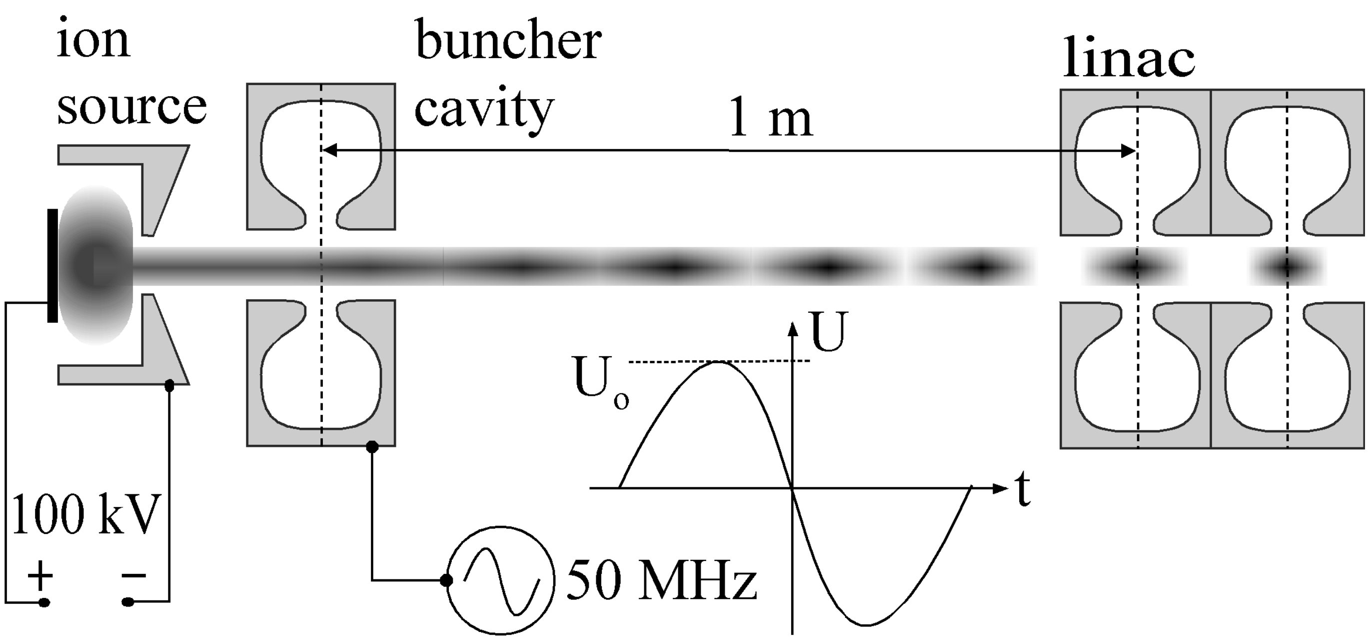
Additional questions:
How long does it take the muons to reach the end of the linac? What is the final kinetic energy?
How does it work with pions ($m_o c^2 = 139.5$ MeV, $q = e$, $\tau = 26$ ns).

Exercise 3

Assume a continuous proton beam from a 100 kV ion source. A buncher cavity operating at a frequency of 50 MHz modulates the energy of the protons. The corresponding modulation of velocities leads to different times of flight to a location further downstream, and thus to a formation of beam bunches from the initially continuous current.

This process is called velocity bunching or ballistic bunching.

At a distance L=1 m after the buncher the bunching should be optimum in order to inject the protons into a linac there (i.e. the bunch should have minimum length there).



Question: What's the required voltage amplitude U_o of the buncher cavity?

Hints:

What's the proton beam velocity? Is it allowed to do a non-relativistic calculation?

Assume that the buncher is "short", i.e. any variation of fields inside the buncher may be neglected.

Calculate time of flight for a proton at time Δt with respect to a reference particle.

Introduce approximations (and check validity!):
a) linearize the time dependance ($\sin \omega t$) of the electric field in the buncher.
b) Assume the energy change due to the buncher small compared to the initial energy.

Exercise 4

A quadrupole doublet (slide 82) provides focusing both horizontally and vertically (if the distance L between the quads is smaller than the focal length f), however the focus is not at the same location for horizontal and vertical.

Consider a modified doublet, where the strength $\pm\frac{1}{f_2}$ of the second quad is not the exact opposite of the first $\mp\frac{1}{f_1}$:

Calculate the focal length f_2 as a function of f_1 and L to obtain a double focus, i.e. horizontal and vertical focus at same location.
What is the distance to this focus?

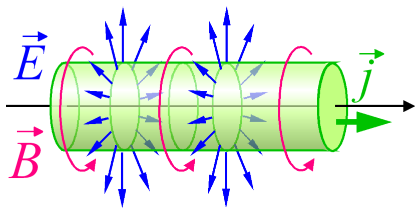
Hint: multiply matrices like shown on slide 82 and find the condition for same focus distances d_x, d_y.

Example: $L = 1$ m, $f_1 = 2$ m $\rightarrow f_2 = 1.5$ m, $d_x = d_y = 3$ m.

Exercise 5

Consider a cylindrical, homogenously charged particle beam of radius R, length L and charge $Q = Ne$, propagating at a velocity $\vec{v} = v\,\hat{\vec{e}}_z$.

A. calculate the force



1. on a particle (charge e, mass m_o) inside inside this beam
2. on an oncoming particle of same velocity but opposite direction, i.e. $\vec{v} = -v\,\hat{\vec{e}}_z$.
3. on an oncoming anti-particle (charge $-e$).

Hints: use Maxwell equations div$\vec{D} = \varrho$ and rot$\vec{H} = \vec{j}$ (with Gauss and Stokes theorems for integral form) to calculate the fields of the bunch. Neglect the ends of the bunch, i.e. assume it as infinitely long. Then the electric field is radial and the magnetic field azimuthal. Make use of $\epsilon_o\mu_o c^2 = 1$. Show that the force on the test particle is radial and linear; compare the result for case 1 to slide 80.

B. calculate the focal length of the lens formed by the oncoming bunch in case 3 for the ultrarelativistic limit with particle energy E.
Put in some numbers: 10^{10} positrons in the bunch, oncoming electrons, both beams at 0.51 GeV (Phi-factory). Radius at interaction point 50 μm. Length of bunch is not needed for calculation!

Hint: assume that the lens is "short", i.e. the coordinate r does not change while the particle travels through the oncoming bunch. Calculate the integrated change of radial momentum for a particle entering with $p_r = 0$, neglect change of longitudinal momentum. It is useful to introduce the "classical electron radius" $r_e = \frac{e^2\mu_o}{4\pi m_o}$.
Focal length f is given by $1/f = kL$ with k, L field gradient and length.

(Result: focal length 4.4 cm.)