Experimental Methods of Particle Physics

Particle Accelerators

Andreas Streun, PSI

andreas.streun@psi.ch

https://ados.web.psi.ch/empp-streun
Particle Accelerators

1. Introduction
2. Accelerator basics and types
3. Single particle dynamics
4. Multi-particle dynamics
5. Longitudinal beam dynamics
6. Synchrotron radiation
7. Luminosity
8. Muons and neutrinos
1. Introduction

- Books & webs
- Why accelerators?
- Particles
  - Particles of interest
  - Particle wavelength and momentum
  - Particles to accelerate
  - Particle production
- A beam of particles
  - Beam quality
  - Accelerator performance
- Particle Physics experiments
  - Center-of-mass energy
  - Luminosity
Books & Webs & Lectures

- Proceedings of The CERN Accelerator School
  [http://cas.werb.cern.ch/cas/](http://cas.werb.cern.ch/cas/)
- Proceedings of the Accelerator Conferences

- ETH 2-semester lecture on accelerators and modeling

<table>
<thead>
<tr>
<th>Nummer</th>
<th>Titel</th>
<th>ECTS</th>
<th>Umfang</th>
<th>Dozierende</th>
</tr>
</thead>
<tbody>
<tr>
<td>402-0777-00L</td>
<td>Particle Accelerator Physics and Modeling I</td>
<td>6 KP</td>
<td>2Y + 1U</td>
<td></td>
</tr>
<tr>
<td>402-0777-00 V</td>
<td>Particle Accelerator Physics and Modeling I</td>
<td></td>
<td>2 Std. Fr 10-12 HIT F 31.2 »</td>
<td>A. Adeimann</td>
</tr>
<tr>
<td>402-0777-00 U</td>
<td>Particle Accelerator Physics and Modeling I</td>
<td></td>
<td>1 Std. Fr 13-14 HIT F 12 »</td>
<td>A. Adeimann</td>
</tr>
</tbody>
</table>
Why accelerators? → I. Applications

Length scale

- 10^{-10} \text{ m} = 1 \text{ Å} / \text{atomic}
- 10^{-15} \text{ m} / \text{nuclear}
- 10^{-18} \text{ m} / \text{electroweak}

Materials Research (MR)
  - Physics
  - Chemistry
  - Biology
  - Pharmacy

Nuclear Physics (NP)

Particle Physics (PP)
  - Energy frontier
  - Particle factories
  - Exotic particles

Particle Accelerators

Medical applications

Industrial applications
Why accelerators?  →  II. Connections

Classical Physics
- Hamiltonian Mechanics
- Electrodynamics

Engineering
- Radio-frequency
- Magnet technology
- Ultra high vacuum
- Mechanical engineering
- Alignment & Survey
- Electronics
- Control systems

Particle Accelerators

Modern Physics
- Quantum mechanics
- Particle Physics

Computing
- High performance computing
- Accelerator design codes
- Digital signal processing s/w
- Application programming
Why accelerators? → III. common PP & MR interests

Particle Physics (PP)
- High energy colliders
- Particle factories
- Neutrino factories
- Linear colliders
- Circular colliders
- High power proton accelerators

Materials Research (MR)
- Spallation neutron sources
- Photon sources
- Storage rings
- Free electron lasers (FEL)

PP & MR scientists: → understand potential and limitations of accelerators
→ help to specify future machines
Particles of interest (PP)

Particle Physics: *interested in all particles!*

presently: particular interest in

- new and unknown particles: 
  \[ W^\pm \ (80.4 \text{ GeV}), \ Z \ (91.2 \text{ GeV}), \ H^0 \ (126 \text{ GeV}), \ ...? \]
  produced in \( e^+e^- \) or \( pp \) or \( p\bar{p} \) collision
  \( \Rightarrow \) highest energies: e.g. LEP, LHC

- meson pairs (e.g. \( K_SK_L, B^0\bar{B}^0 \)) at high rate
  \( \Rightarrow \) meson factories: e.g. KEK-B, DAΦNE.

- muons (\( \mu^\pm \)) and neutrinos (\( \nu_e, \nu_\mu \))
  \( \Rightarrow \) long baseline experiments: e.g. CNGS, JPARC
  \( \Rightarrow \) muon accelerator and neutrino factory projects
Particles of interest (MR)

Materials Research: neutral particles
⇒ high penetration depth in materials

♦ Neutrons ($n$)
  ■ penetrate high Z materials
  ■ depth not a steep function of Z
  ■ have a magnetic moment and a spin
  ■ explore structure and dynamics of materials
  ■ rather low flux (= particles per time and area)

♦ Photons ($\gamma$)
  ■ available at (very!) high flux
  ■ penetrate well low Z materials
  ■ have polarization
  ■ complementary to neutrons (“surface vs. bulk”)
Particle wavelength

Size of structure $\Leftrightarrow$ Size of probe

- **MR** $\Rightarrow \lambda \sim 10^{-10}$ m
- **NP** $\Rightarrow \lambda \sim 10^{-15}$ m
- **PP** $\Rightarrow \lambda \sim 10^{-18}$ m

**De-Broglie wavelength**

$$\lambda = \frac{h}{p}$$

**Planck constant**

$$h = 6.63 \cdot 10^{-34} \text{ Js} = 4.14 \cdot 10^{-15} \text{ eV} \cdot \text{s}$$

**Particle momentum**

$$\begin{align*}
p &= m \cdot v = m \cdot c \cdot \beta \gamma \\
\text{non-relativistic} &\quad p = m \cdot v \\
\text{high relativistic} &\quad p = m \cdot c \cdot \gamma = E / c
\end{align*}$$
Recall: momentum & energy

Momentum

- $p = m \cdot v = m_o c \cdot \beta \gamma$
- non-relativistic $p = m_o v$
- high relativistic $p = m_o c \cdot \gamma = E/c$

Total energy

$E = mc^2 = m_o c^2 \gamma = \sqrt{(m_o c^2)^2 + (pc)^2} = E_{kin} + m_o c^2$

Kinetic energy

- $E_{kin} = m_o c^2 (\gamma - 1) = q \cdot U = \text{charge} \times \text{voltage}$.
- $E_{kin}$ in units of eV is equivalent to the accelerating voltage for a particle with charge $q = 1e$ ($p$, $e^+$, $Na^+$, $\mu^+$...)
- non-relativistic $E_{kin} = \frac{1}{2} m_o v^2$
- high relativistic $E_{kin} = pc$

useful relations:

$\beta = \sqrt{1 - \frac{1}{\gamma^2}} \quad \gamma = \frac{1}{\sqrt{1 - \beta^2}} \quad \beta \gamma = \sqrt{\gamma^2 - 1}$
Examples: momentum and wavelength

**MR: 1Å neutron** \((m_o c^2 = 940.8\text{ MeV}, m_o = 1.68 \cdot 10^{-27}\text{ kg})\)
- \(p = 12.4\text{ keV}/c\)
- \(v = 3960\text{ m/s} \ll c\)
- \(E_{\text{kin}} = 0.08\text{ eV} \Rightarrow <E_{\text{kin}}> = kT \Rightarrow \)
- temperature equivalent \(T = 930\text{ K}\)

**MR: 1Å photon** (no rest mass)
- \(v = c, \ E_\gamma = pc = 12.4\text{ keV} \rightarrow \text{ X-ray}\)

**NP: 10^{-15} m electron** \((m_o c^2 = 511\text{ keV})\)
- \(p = 1.24\text{ GeV}/c\)
- \(v = 0.999’999’915\ c = c – 90\text{ km/h}!\)
- \(E_{\text{kin}} = pc = 1.24\text{ GeV}\)

**PP: 10^{-18} m proton** \((m_o c^2 = 938.3\text{ MeV})\)
- \(p = 1.24\text{ TeV}/c\)
- \(v = 0.999’999’7\ c\)
- \(E_{\text{kin}} = pc = 1.24\text{ TeV} \ (\Rightarrow \text{LHC 7 TeV})\)
Particles to accelerate

Requirements for acceleration:

charge \( q \neq 0 \) and lifetime \( \tau \geq \approx 1 \ \mu s \)

- standard: electron \( e^- \) and proton \( p \)
- antiparticles: positron \( e^+ \) and antiproton \( \bar{p} \)
- ions: \( \frac{A}{Z} X q/e \): \( \frac{1}{1} H^+ = p, \frac{1}{1} H^-, \frac{4}{2} He^{2+} = \alpha \ldots \frac{238}{92} U^{n+} \ldots \)
- muons: \( \mu^+ \), \( \mu^- \) \( \tau = 2.2 \ \mu s \)
- pion \( \pi^\pm \) (\( \tau = 26 \ \text{ns} \)), neutron \( n \), neutrino \( \nu \), photon \( \gamma \) ...
## Particle Production

how to get the particles of interest from the particles that can be accelerated

<table>
<thead>
<tr>
<th>Principle</th>
<th>Products</th>
<th>Performance</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Beam on target</strong></td>
<td></td>
<td>Flux↓ chapter 8</td>
</tr>
<tr>
<td>electrons (e^-)</td>
<td>pairs (e^+e^-, pp)</td>
<td></td>
</tr>
<tr>
<td>protons (p)</td>
<td>mesons (\pi \rightarrow \mu \rightarrow \nu)</td>
<td></td>
</tr>
<tr>
<td>spallation target</td>
<td>neutrons (n)</td>
<td></td>
</tr>
<tr>
<td><strong>Colliding beams</strong></td>
<td></td>
<td>Luminosity↓ chapter 7</td>
</tr>
<tr>
<td>leptonic (e^+e^-)</td>
<td>anything...</td>
<td></td>
</tr>
<tr>
<td>hadronic (pp)</td>
<td>mesons (KK, BB)...</td>
<td></td>
</tr>
<tr>
<td>Higgs (H^0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Synchrotron radiation</strong></td>
<td></td>
<td>Brightness↓ chapter 6</td>
</tr>
<tr>
<td>electrons (e^-)</td>
<td>photons (\gamma)</td>
<td></td>
</tr>
</tbody>
</table>
A beam of particles

Particle beam \((n, \mu, \gamma, e^-, p \ldots)\)

= ensemble of \(N\) particles in 6-dimensional phase space \((x, y, z; p_x, p_y, p_z)\)

1\(^{\text{st}}\) order

Beam centroid
mean values \(< r_i >\)
- beam momenta \(p_x, p_y, p_z\)
  moving along “z”
  \(\rightarrow p_z \approx p \gg p_x, p_y\)
- beam location \(z(t)\)
- beam positions \(x, y\)
- beam angles \(x' \approx p_x/p, y'\)

2\(^{\text{nd}}\) order

Beam distribution
rms values \(\sigma_i^2 = < r_i^2 >\)
and correlations \(< r_i r_j >\)
- momentum spread \(\sigma_{\Delta p/p}\)
- “bunch length” \(\sigma_{\Delta z}\)
- beam sizes \(\sigma_x, \sigma_y\)
- beam divergences \(\sigma_{x'}, \sigma_{y'}\)
- ... correlations ...
Beam quality $\rightarrow$ I. phase space density

Criterion for beam quality $(n, \nu, \gamma, e^-, p, ...)$:

- **density** in 6-d phase space

$\Rightarrow$ performance of experiment

$\rightarrow$ flux, luminosity, brightness,

$\Rightarrow$ threshold phenomena

$\rightarrow$ coherence, non-linearity...

**Theorem of Liouville:**

(holds under several conditions....)

"The 6-d phase space density is an invariant." or

“The 6-d phase space occupied by a beam behaves like an incompressible liquid.”
Beams quality $\rightarrow$ II. Emittance

Decoupling of 6-d phase space density into $3 \times 2$ dimensions (this is often $\approx$ possible):

- Longitudinal $\times$
  - $\Delta p/p, \Delta z$
  - Momentum spread
  - Pulse (bunch*) length

- Horizontal $\times$
  - $x, p_x$ (or $x'$)
  - Transverse emittances $\varepsilon_x, \varepsilon_y$

- Vertical
  - $y, p_y$ (or $y'$)

2-d phase space area:

$$\varepsilon_x^2 = \langle x^2 \rangle \langle x'^2 \rangle - \langle xx' \rangle^2$$

Invariant along beam transport line

$\Rightarrow$ chapter 4

* beams are usually “bunched”, not continuous

$\Rightarrow$ chapter 2
Beam quality → III. Accelerator performance

- Momentum
  (high relativistic: energy $E = pc$)
- Momentum spread $\sigma_{Ap/p}$
- Bunch length $\sigma_{Az}$
- Emittances $\varepsilon_x, \varepsilon_y$
- Beam current $I = q \cdot dN/dt$
- Higher orders...
  (non-Gaussian, halo, tails etc.)
- Polarization (spin orientation)
- Time structure: [continuous or] “bunched” → repetition rate
- Stability: position, angle, momentum, timing...
  jitter as function of frequency

6-d phase space density

Experiment performance:
Luminosity (PP)
Brightness (MR)
PP-experiment → I. Center-of-mass energy

Beam on target

\[ \vec{p}_2 = 0 \quad E_1 = E \]

\[ E_{\text{cm}} \approx \sqrt{2E m_2 c^2} \]

Colliding beams

\[ \vec{p}_2 = -\vec{p}_1 \quad E_1 = E_2 = E \]

\[ E_{\text{cm}} = 2E \]

Beam on target

- Antiparticles: \( e^+ \), \( \bar{\rho} \)
- Mesons for experiments: \( \pi \), \( K \) . . .
- Muons and neutrinos: \( \mu \) \[ \rightarrow \nu_\mu \]
- Spallation neutrons: \( n \)

Colliding beams

- possibility of pure leptonic production \( e^+ \rightarrow e^- \)
- highest energies (e.g. LHC \( E_{\text{cm}} = 14 \) TeV)
- high luminosity particle factories: \( \Phi \), \( B \) . . .
- variable boost \( \vec{\beta} \) from \( E_1 \neq E_2 \) (→ B-factories)
PP-experiment $\rightarrow$ II. Luminosity

Layout of experiment

- Required energy $E_{\text{cm}}$ [and boost $\vec{\beta}$] $\rightarrow$ $E_1$, $E_2$
- Required precision $p$ $\rightarrow$ Events $N \approx \frac{1}{p^2}$ \( p = \frac{\Delta N}{N} \) $\Delta N \approx \sqrt{N}$
- Efficiency of experiment $\epsilon$ (Observation inside detector, branching ratio etc.)
- Scheduled time of measurement $T$ [s]

\[ \Rightarrow \text{Particle production rate} \quad R \ [s^{-1}] = \frac{N}{\epsilon T} \]

- Production cross section $\sigma$ [cm$^2$, or barn $= 10^{-24}$ cm$^2$]

\[ \Rightarrow \boxed{R = \sigma \mathcal{L}} \quad \textit{Luminosity} \quad \mathcal{L} \ [\text{cm}^{-2}\text{s}^{-1}] \]
Luminosity = sum of all possible encounters per time and area

\[ \mathcal{L} = \frac{N_1}{T} \times \frac{N_2}{A^*} \]

\((A^* \text{ common interaction area})\)

\[ \mathcal{L} = \text{particle current of beam 1} \times \text{particle density of beam 2} \]

Requirements for PP-machines

- high beam currents: \(N_1, N_2, T\)
- focus at collision: \(A^*\)
- (highest \(\leftrightarrow\) given) beam energies: \(E_1, E_2 \quad \rightarrow \quad E_{cm} \left[\nu, \beta\right]\)

\(\Rightarrow\) challenges for accelerators.
"Livingston plot"

Progress of center of mass energy

M. L. Tigner, Does accelerator based particle physics have a future? Physics Today 54.1 (2001)