6. Synchrotron Radiation

- **Synchrotron radiation**

  Power. Collimation. Time structure. Spectrum. Brightness.

- **Storage Ring based Light Sources**

  Radiation equilibrium. Swiss Light Source SLS at PSI.
  The SLS-2 upgrade project.

- **Free Electron Lasers**

  FEL schemes. X-ray FEL layout. SwissFEL at PSI.

Lorentz transformation

Transformation from lab sytem K to system K' moving at speed βc in z-direction:

$$\begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix} = M_L \cdot \begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix} \qquad \begin{pmatrix} p_x' \\ p_y' \\ p_z' \\ E'/c \end{pmatrix} = M_L \cdot \begin{pmatrix} p_x \\ p_y \\ p_z \\ E/c \end{pmatrix}$$

$$M_L = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & -\beta\gamma \\ 0 & 0 & -\beta\gamma & \gamma \end{pmatrix} \qquad M_L^{-1} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & +\beta\gamma \\ 0 & 0 & +\beta\gamma & \gamma \end{pmatrix}$$

Moving particle: $z' = 0 \to z = \beta\gamma ct'$ and $ct = \gamma ct'$ $\to$ lab system: $z = \beta ct$

4-vectors: space-time $\tilde{S} = (x, y, z, ict)$ and momentum-energy $\tilde{P} = (p_x, p_y, p_z, iE/c)$

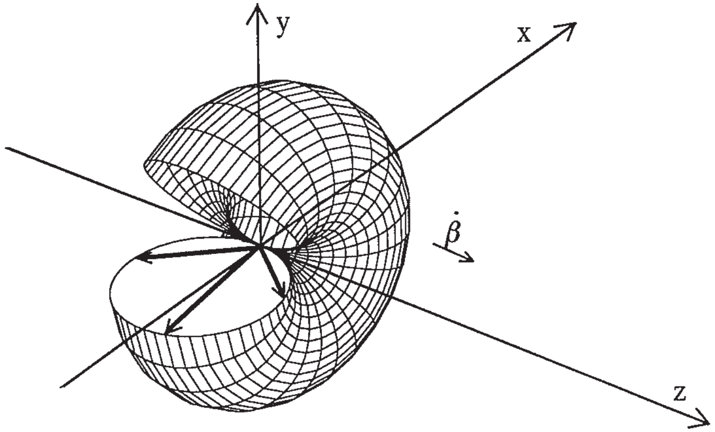
Length of 4-vectors is Lorentz-invariant. $|\tilde{P}| = \sqrt{\tilde{P} \cdot \tilde{P}} = im_oc$

Synchrotron radiation power

Radiation of an accelerated charged particle (Hertz dipole characteristics):

Larmor formula, sub-relativistic

$$P = \left(\frac{e}{m_o c^2}\right)^2 \frac{c}{6\pi\varepsilon_o} \left(\frac{d\vec{p}}{dt}\right)^2 \qquad \text{[SI]}$$



Angular distribution $\frac{d^2 P}{d\phi\, d\theta} \sim \sin^2\theta$

Maximum power $\perp$ to acceleration

Relativistic invariant formulation using 4-momentum $\tilde{P} = [\vec{p},\, iE/c]$

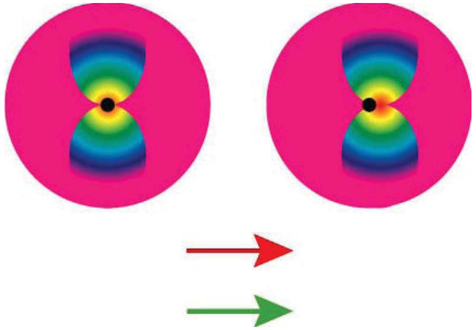
4-D scalar product: $\tilde{P}_a \cdot \tilde{P}_b = \vec{p}_a \cdot \vec{p}_b - E_a E_b / c^2 \quad \longrightarrow \quad \tilde{P}^2 = -m_o c$

$$\left(\frac{d\vec{p}}{dt}\right)^2 \quad \longrightarrow \quad \left(\frac{d\tilde{P}}{dt'}\right)^2 = \left(\frac{d\vec{p}}{dt'}\right)^2 - \frac{1}{c^2}\left(\frac{dE}{dt'}\right)^2 \qquad \text{with } t' = \frac{1}{\gamma}\, t \ \text{ time in moving system.}$$

consider $d\vec{p}/dt' \parallel \vec{p}$ linear acceleration → linac

$d\vec{p}/dt' \perp \vec{p}$ circular acceleration → synchrotron

Linear acceleration



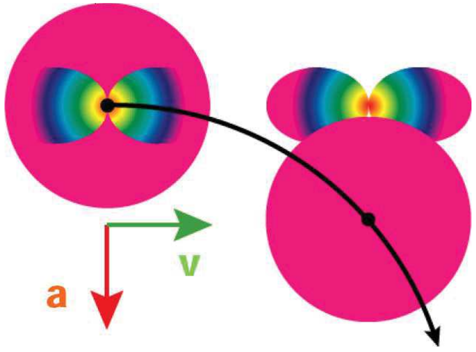
Radiation cannot separate from the Coulomb field.

$E^2 = (m_o c^2)^2 + (pc)^2 \rightarrow \frac{dE}{dt'} = \beta c \frac{dp}{dt'} \qquad (1-\beta^2) = 1/\gamma^2$

$\left(\frac{d\tilde{P}}{dt'}\right)^2 = \left(\frac{d\vec{p}}{dt}\right)^2 = (e\vec{E})^2$ (electric field)

Example: acceleration with gradient $|\vec{E}| = 25\ \mathrm{MV/m} \quad \longrightarrow \quad P = 10^{-16}\ \mathrm{W}$
per $1\ \mathrm{m}$ linac: electron energy increase $25\ \mathrm{MeV}$, radiation loss $2\ \mu\mathrm{eV}$ → ***negligible!***

Circular acceleration



Radiation separates fast from the Coulomb field.

$dE/dt' = 0 \quad \longrightarrow \quad \frac{d\vec{p}}{dt'} = \gamma \frac{d\vec{p}}{dt}$

centrifugal acceleration $\frac{dp}{dt} = \frac{mv^2}{R} = \frac{pv}{R} = \frac{\beta pc}{R} = \frac{\beta^2 E}{R}$

$$\left(\frac{d\tilde{P}}{dt'}\right)^2 = \left(\frac{\beta^2 \gamma E}{R}\right)^2 \quad \longrightarrow \quad P = \frac{e^2 c}{6\pi\varepsilon_o}\left(\frac{E}{m_o c^2}\right)^4 \frac{\beta^4}{R^2} \qquad [\mathrm{SI}]$$

Energy loss per turn $U_o = P\,\frac{2\pi R}{c}$ (radiation only in bending magnets)

$$\beta \approx 1 \quad \longrightarrow \quad U_o\ [\mathrm{keV}] = \underbrace{10^{33}\,\frac{e}{3\varepsilon_o}\left(\frac{e}{m_o c^2}\right)^4}_{88.5} \frac{(E\ [\mathrm{GeV}])^4}{R\ [\mathrm{m}]}$$

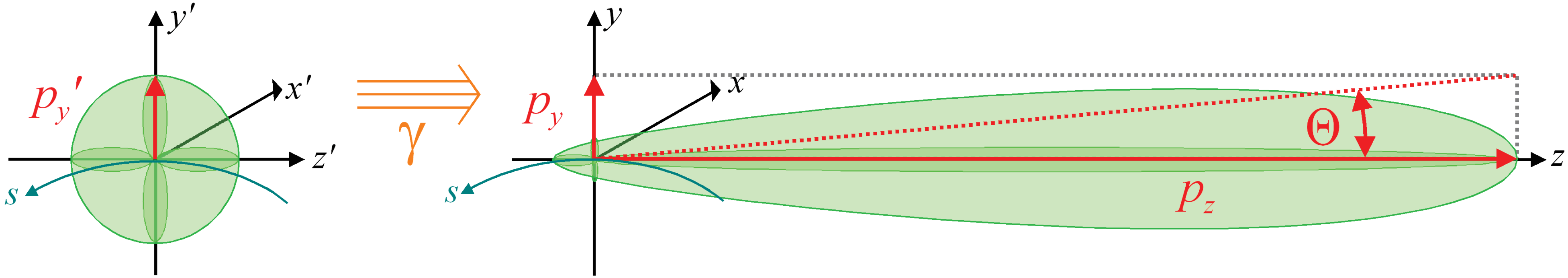
Example: **SLS** at $2.4\ \mathrm{GeV}$, $R = 5.7\ \mathrm{m} \quad \longrightarrow \quad U_o = 512\ \mathrm{keV}$ per electron.
max. current $I = 400\ \mathrm{mA} \rightarrow P = U_o \cdot I = 205\ \mathrm{kW}! \rightarrow$ supplied by RF.

Collimation

Acceleration in x-direction $\longrightarrow$ max. emission in y and z directions.

Assume photon ($\beta = 1!$) momentum in y direction: $p'_y = \frac{E'}{c}$, $p_z = 0$

Lorentz transformation to lab system: $p_y = p'_y \qquad p_z = \gamma \frac{E'}{c} = \gamma p'_y$

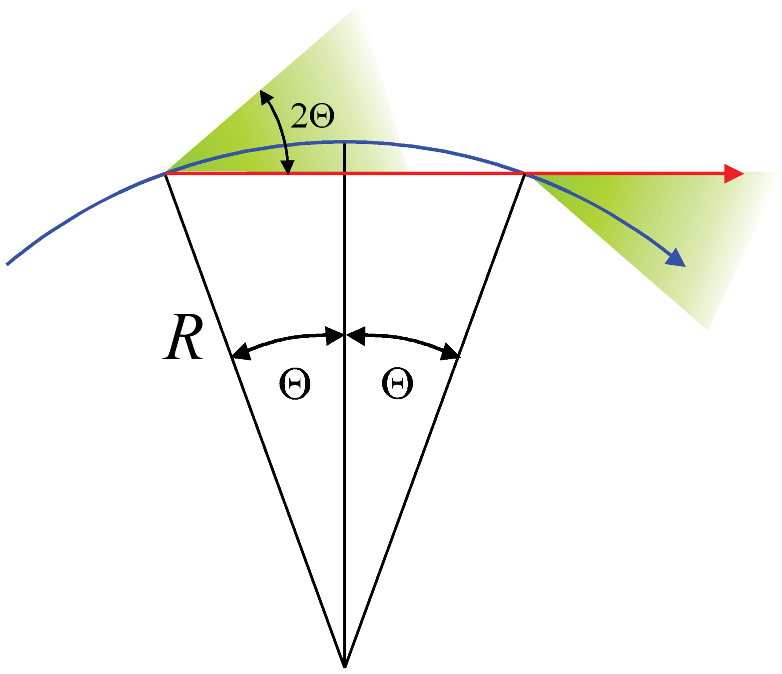


Collimation angle $\tan\Theta = \frac{p_y}{p_z} = \frac{1}{\gamma}$

Example: **ESRF** at 6 GeV $\longrightarrow$ $\Theta = 85\,\mu$rad.
Beam spot 1 cm diameter in 60 m distance.

ESRF (**E**uropean **S**ynchrotron **R**adiation **F**acility) (Grenoble, France) ⟶

Time structure and photon energy



Collimation → Observation from narrow sector (small *depth of field*)

pulse duration = time delay (electron – photon)

$$\Delta t = \frac{2R\Theta}{c\beta} - \frac{2R\sin\Theta}{c}$$

$$\longrightarrow \quad \sin\Theta \approx \Theta - \tfrac{\Theta^3}{6}, \qquad \tfrac{1}{\beta} = \tfrac{1}{\sqrt{1-1/\gamma^2}} \approx \tfrac{1}{1-1/(2\gamma^2)} \approx 1 + \tfrac{1}{2\gamma^2} \quad \longrightarrow \quad \Delta t = \frac{4R}{3c\gamma^3}$$

$\Longrightarrow$ typical frequency $\nu_{\text{typ}} = \frac{1}{\Delta t}$ and energy $\tilde{E}_{\text{typ}} = h\nu_{\text{typ}} = \frac{3hc}{4R}\gamma^3$.

Example: ESRF at 6 GeV, $R = 23$ m $\longrightarrow$ $\tilde{E}_{\text{typ}} = 65$ keV – ***like X-ray tube***

Radiation spectrum

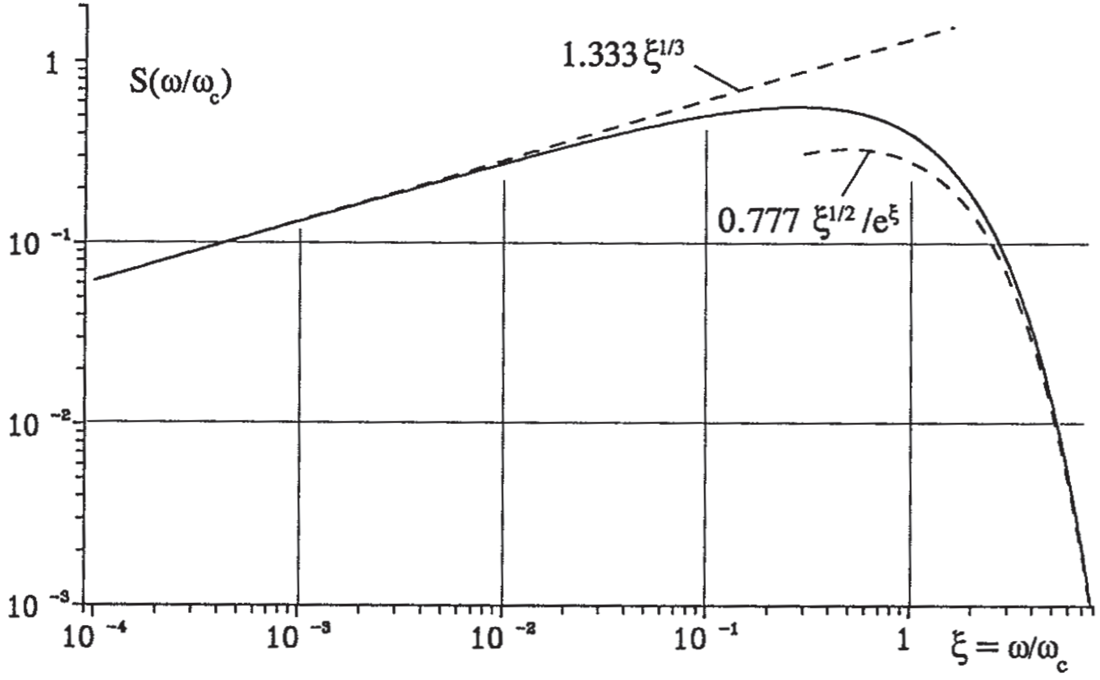


Fig. 7.12. Universal function: $S(\omega/\omega_c) = \frac{9\sqrt{3}}{8\pi}\frac{\omega}{\omega_c}\int_{\omega/\omega_c}^{\infty} K_{5/3}(x)\,dx$

(figure from: H.Wiedemann, *Particle accelerator physics 2*)

$$\frac{dP}{d\omega} = \frac{P}{\omega_c} S\left(\frac{\omega}{\omega_c}\right) \qquad (\tilde{E} = h\omega)$$

Define *critical energy (frequency)*

$\tilde{E}_c\ (\omega_c)$:

$$\int_0^{\omega_c} \frac{dP}{d\omega}\,d\omega \overset{!}{=} \int_{\omega_c}^{\infty} \frac{dP}{d\omega}\,d\omega$$

$$\tilde{E}_c = h\omega_c = \frac{1}{\pi}\,\tilde{E}_{\mathrm{typ}}$$

$\longrightarrow$ use $BR = p/e$ $\longrightarrow$

$$\tilde{E}_c\ [\mathrm{keV}] = \underbrace{10^{15}\,\frac{3hc^2}{4\pi e}\left(\frac{e}{m_o c^2}\right)^3}_{0.665} B\ [\mathrm{T}]\ (E\ [\mathrm{GeV}])^2$$

Angular and energy distribution

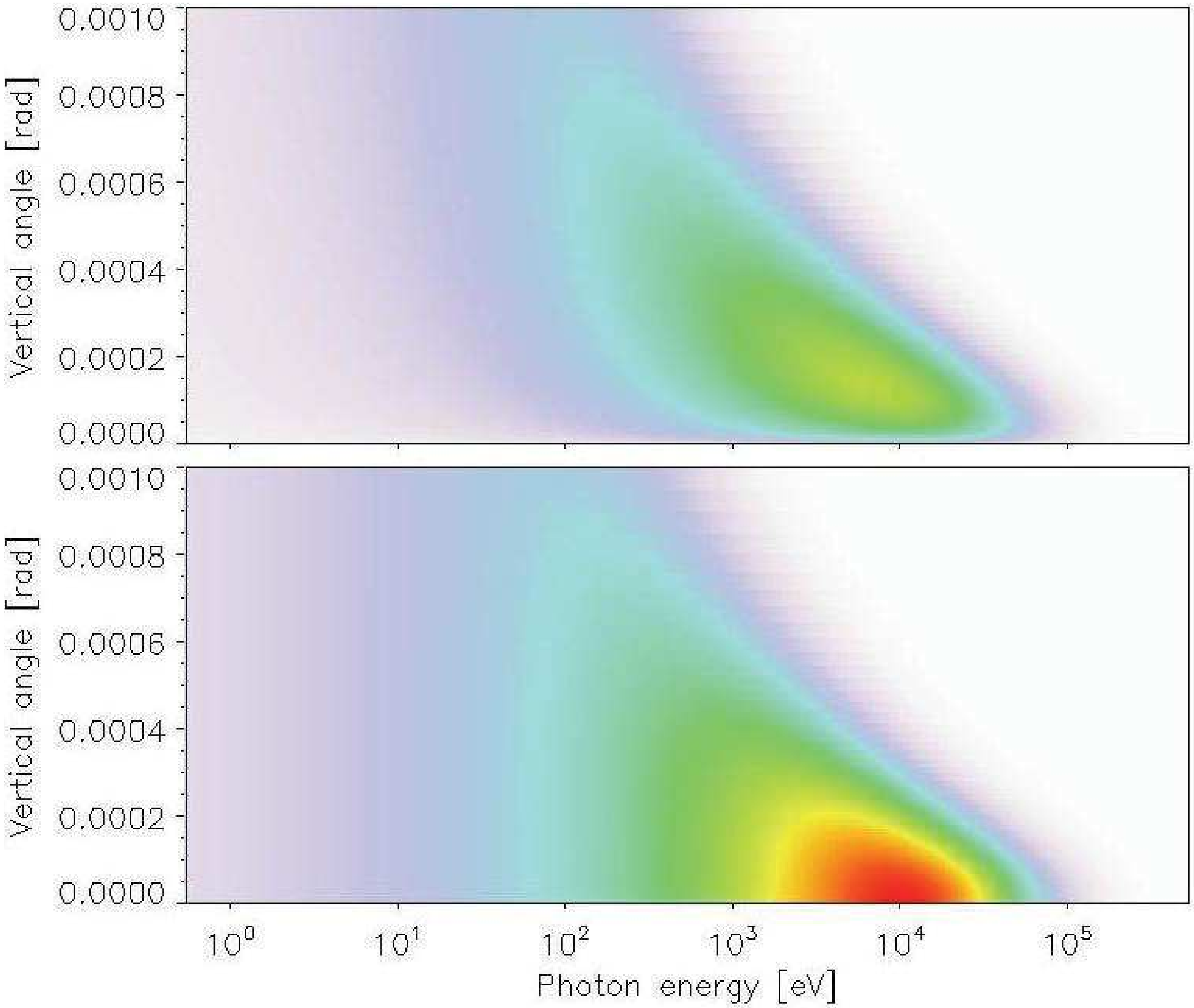
Power density

vertical polarization
(1/8 of total power)

horizontal polarization
(7/8 of total power)

SLS bending magnet

$1.4\ \mathrm{T}, 2.4\ \mathrm{GeV}$
$\rightarrow \tilde{E}_c = 5.4\ \mathrm{keV}$



Brightness and Undulators

Brightness $\mathcal{B}$ = 6-d phase space photon density = spatial and angular flux density

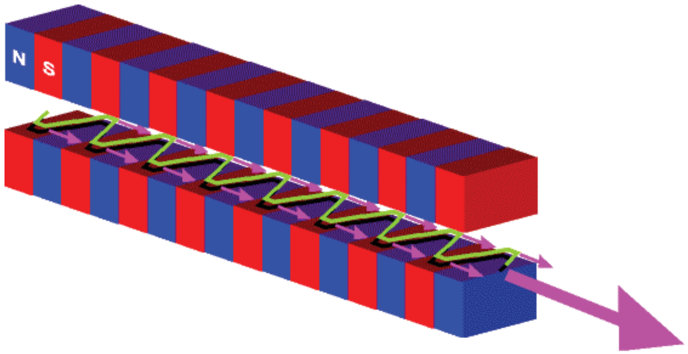
$[\mathcal{B}] = \frac{\text{photons}}{\text{s}\,\text{mm}^2\,\text{mrad}^2\,0.1\%\,\text{BW}}$ $\quad$ $\text{BW} =$ bandwidth $\frac{\Delta\tilde{E}}{\tilde{E}}$ (usually 0.1%)

$\mathcal{B} \sim \frac{1}{\epsilon_x\,\epsilon_y}$ $\longrightarrow$ Light sources require low transverse emittances.

Example **SLS** $\epsilon_x = 5 \cdot 10^{-9}\ \text{rad m},\ \epsilon_y \approx 5 \ldots 10 \cdot 10^{-12}\ \text{rad m}$

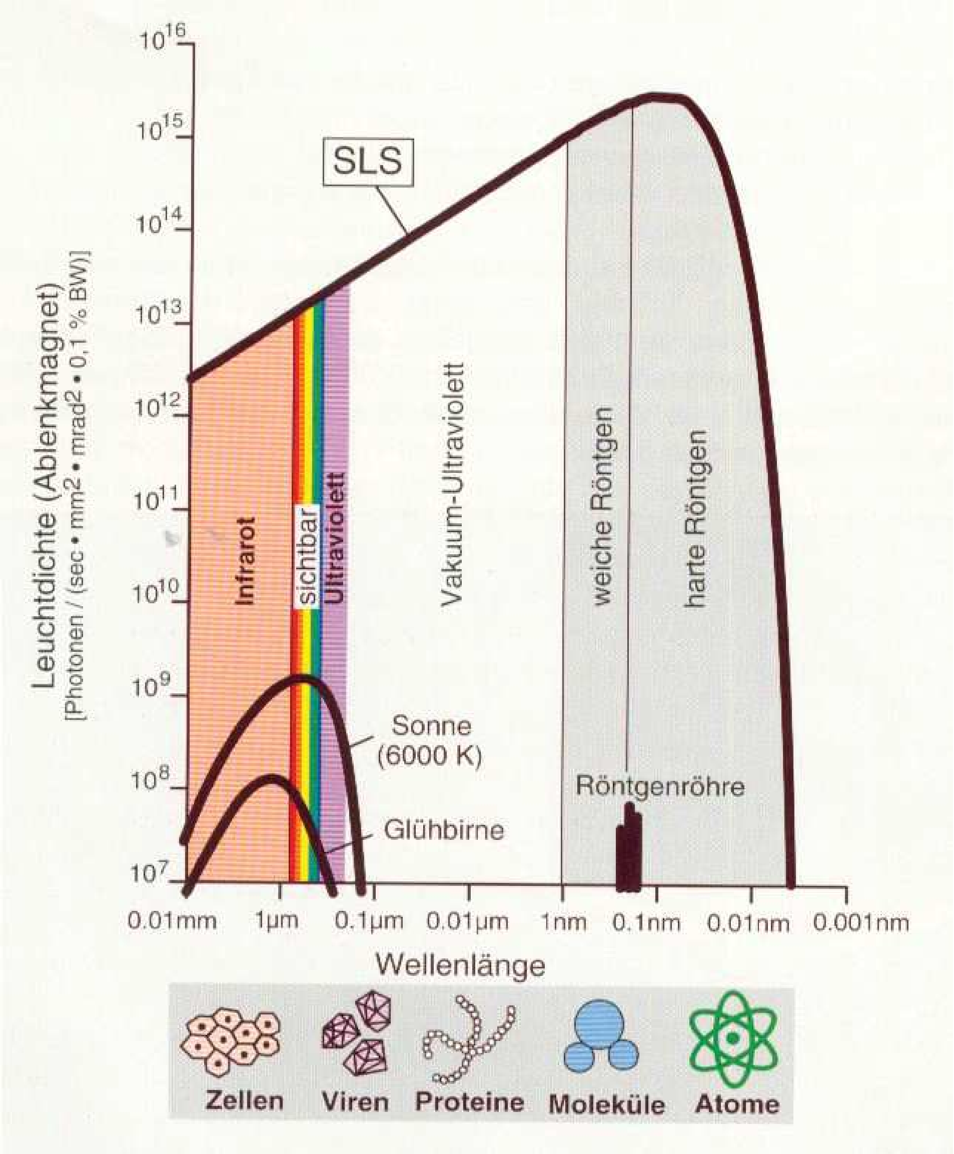
$\longrightarrow$ source size $\sigma_x = 45 \ldots 160\ \mu\text{m},\ \sigma_y = 2 \ldots 8\ \mu\text{m}$ (for different locations)

Undulator magnet

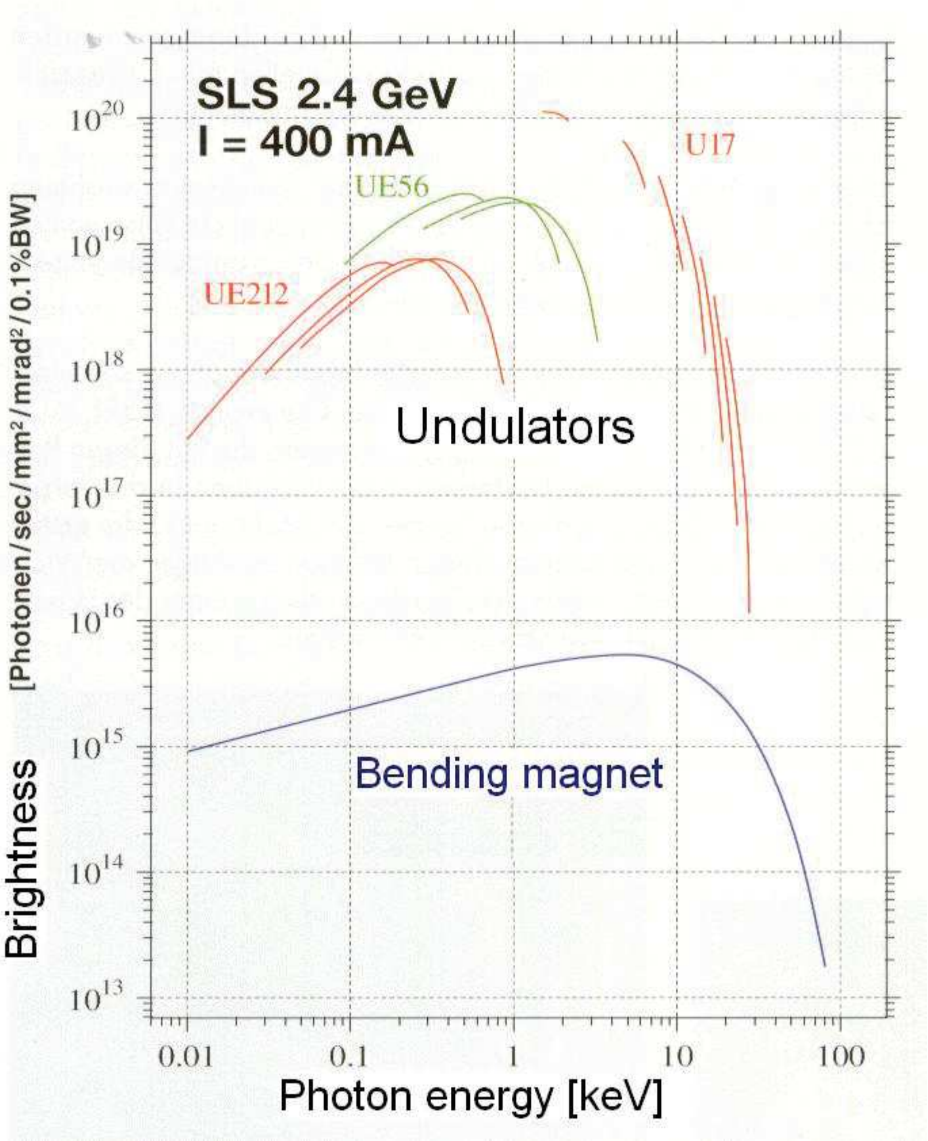


→ coherent superposition of radiation
→ line spectrum
→ very high brightness

SLS *brightness*



Bending magnet brightness in comparison to light bulb, sun and X-ray tube



Undulator brightness in comparison to bending magnet brightness

The radiation equilibrium

Horizontal emittance in electron storage ring:

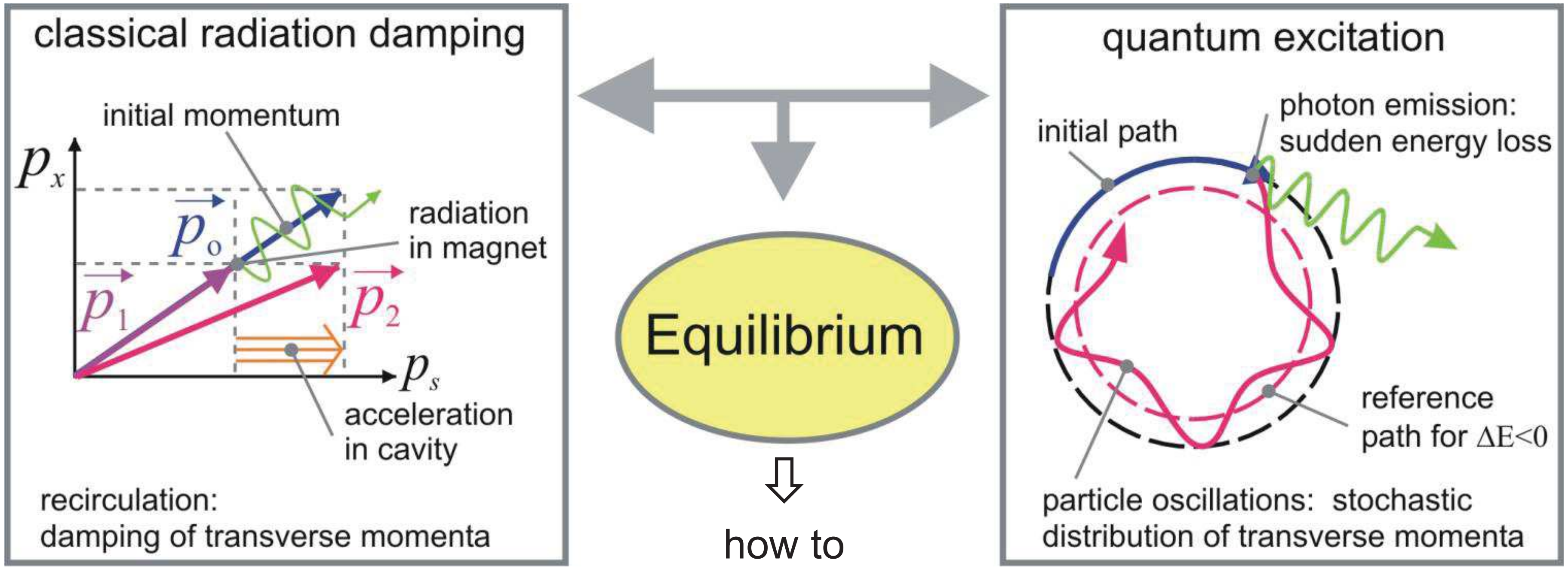
↓radiation damping↓ $\Rightarrow$ **equilibrium** $\Leftarrow$ ↑quantum excitation↑

independent from initial conditions !

double violation of Liouville’s theorem

friction (damping)

stochastic single particle effect



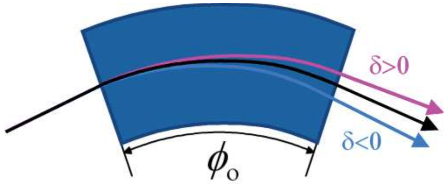
⇧ maximize this -- and -- minimize this ⇧

?

How to minimize storage ring emittance

- Maximum radiation damping (✓)
  - increase radiated power ⇨ pay with RF-power
    - High field bending magnets? ⇨ quantum excitation higher too ✗
    - **Damping wiggler**: Σ |deflection angles| $> 360°$ (✓)
- Minimum quantum excitation ✓
  - keep off-momentum orbit close to nominal orbit

$$\text{Dispersion} = \frac{\text{orbit}}{\text{momentum}} = \frac{X}{\Delta p / p}$$

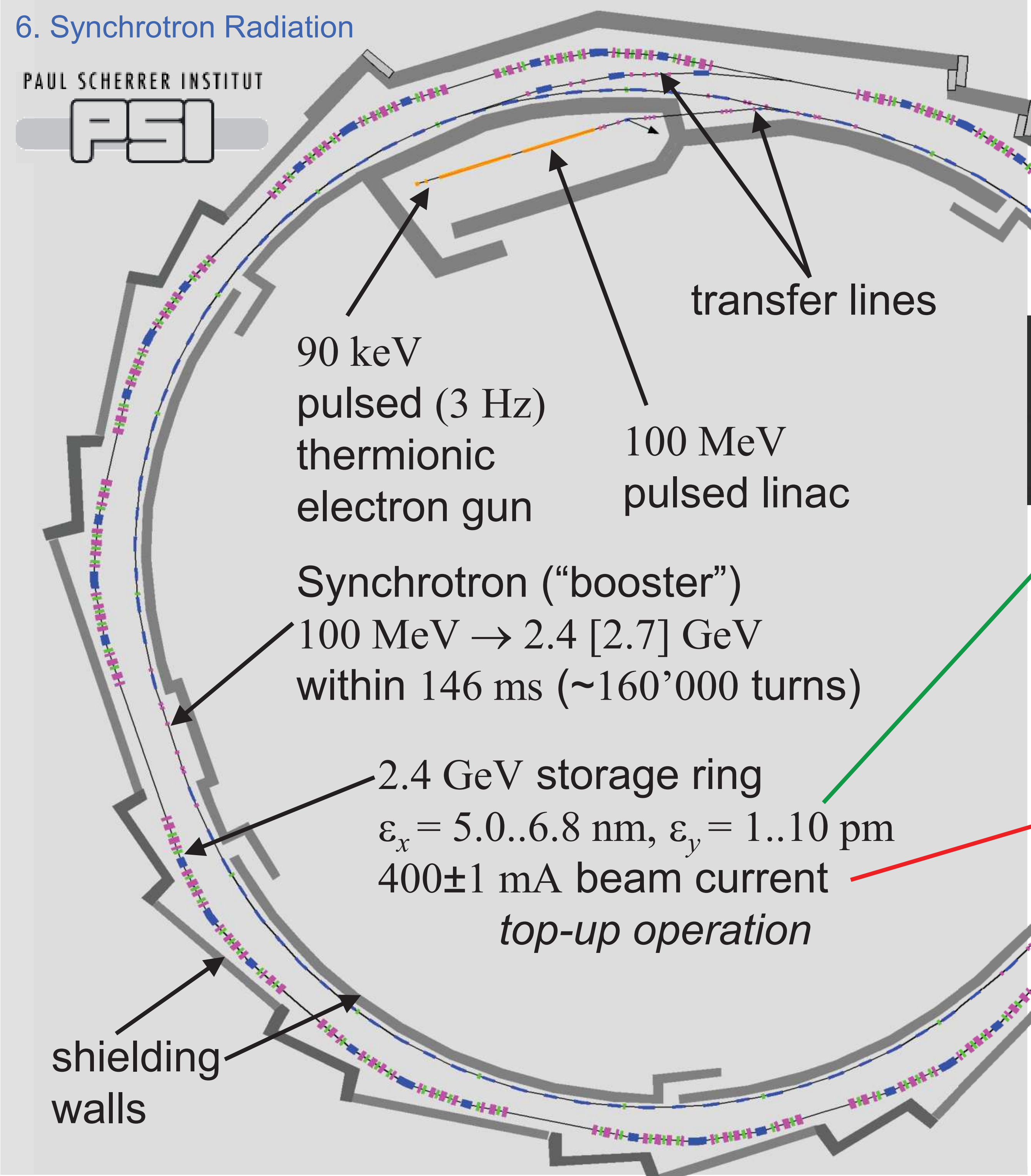


$\delta = \Delta p / p$

⇨ minimize dispersion at locations of radiation (bends)

- **Horizontal focusing** into bends to suppress dispersion. ✓
- **Multi-Bend Achromat** (MBA)
  many short (= low angle) bends to limit dispersion growth. ✓
- **Longitudinal Gradient Bend** (LGB)
  highest radiation at region of lowest dispersion and v.v. ✓

Swiss Light Source SLS



transfer lines

90 keV
pulsed (3 Hz)
thermionic
electron gun

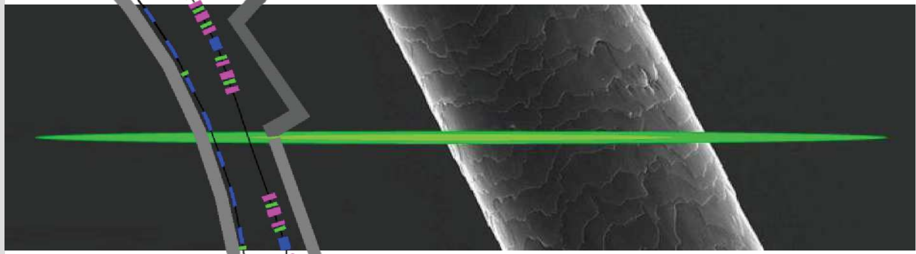
100 MeV
pulsed linac

Synchrotron (“booster”)
100 MeV → 2.4 [2.7] GeV
within 146 ms (~160’000 turns)

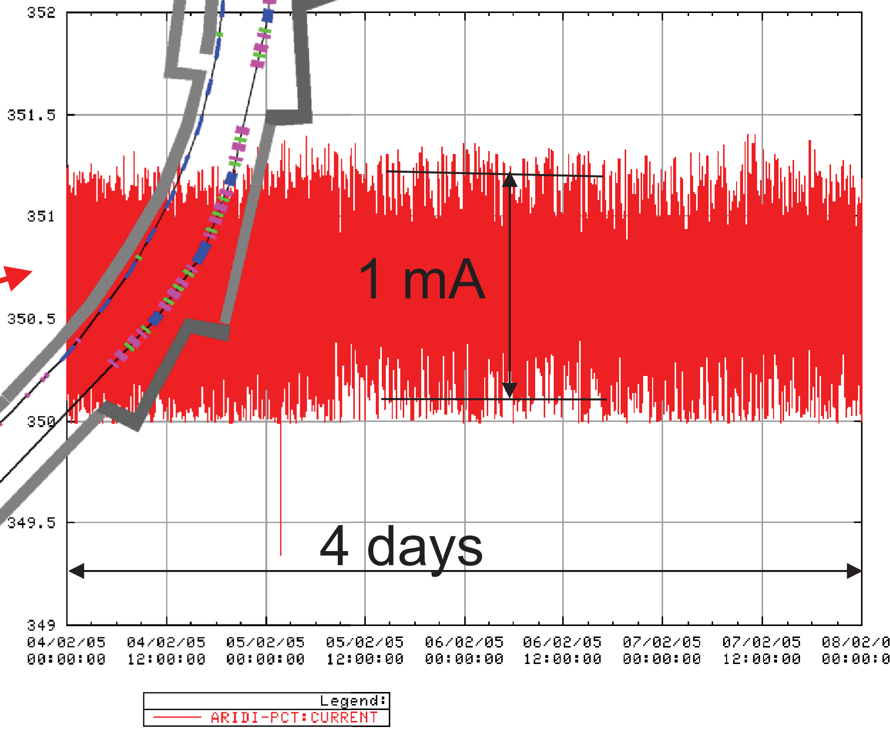
2.4 GeV storage ring
$\varepsilon_x = 5.0..6.8$ nm, $\varepsilon_y = 1..10$ pm
400±1 mA beam current
top-up operation

shielding
walls

Electron beam cross
section in comparison to
human hair

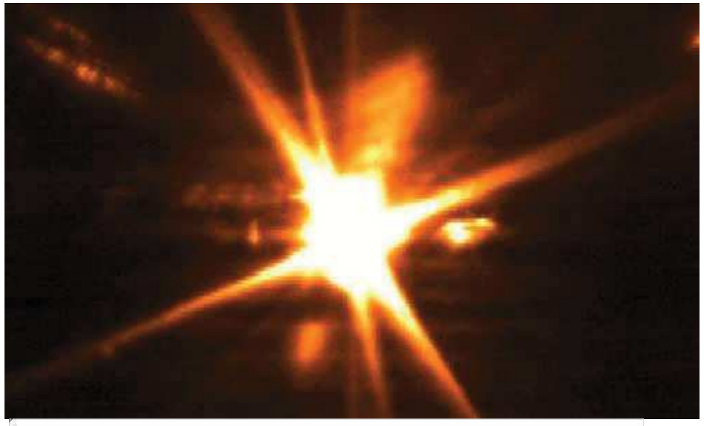


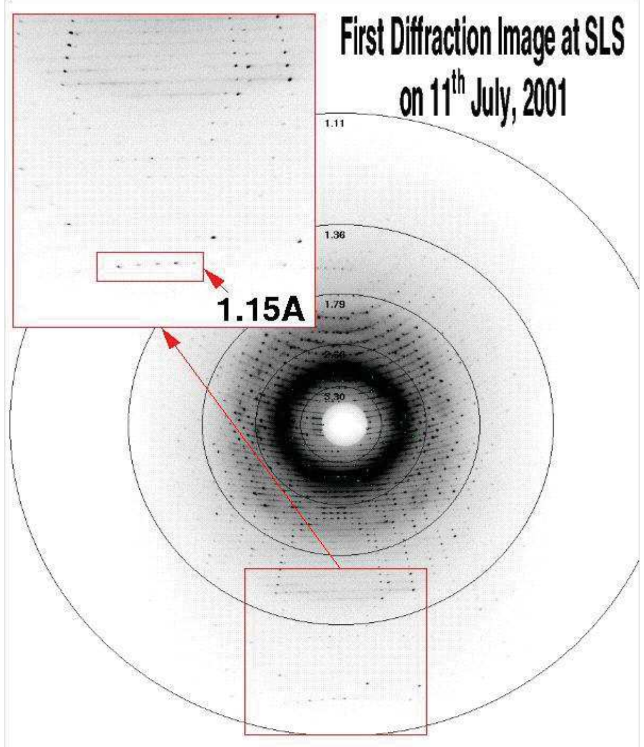
Current vs. time



SLS history

	1990	First ideas for a **Swiss Light Source**
	1993	Conceptual **Design** Report
June	**1997**	**Approval** by Swiss Government
June	**1999**	Finalization of **Building**
Dec.	**2000**	First **Stored Beam**
June	**2001**	Design current **400 mA** reached **Top up** operation started
July	**2001**	First **experiments**
Jan.	**2005**	**Laser beam slicing “FEMTO”**
May	**2006**	**3 Tesla super bends**
	2010	**Completion: 18 beamlines**

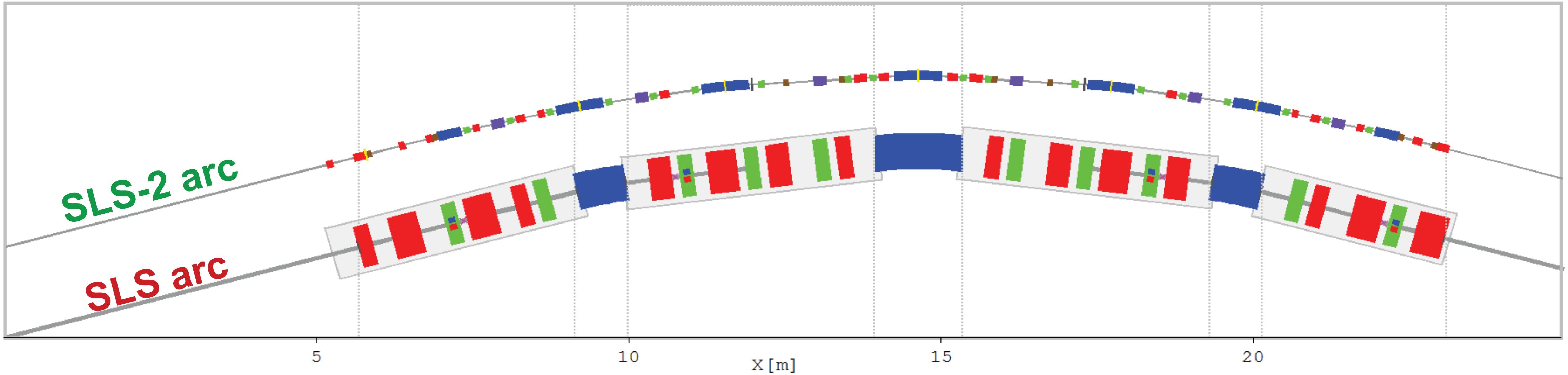


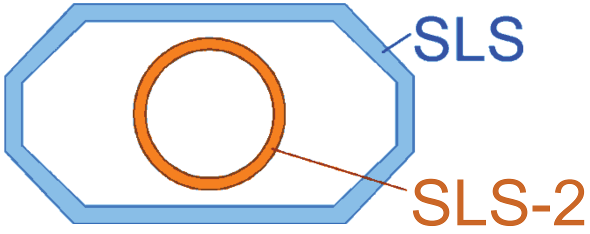


First Diffraction Image at SLS on 11th July, 2001

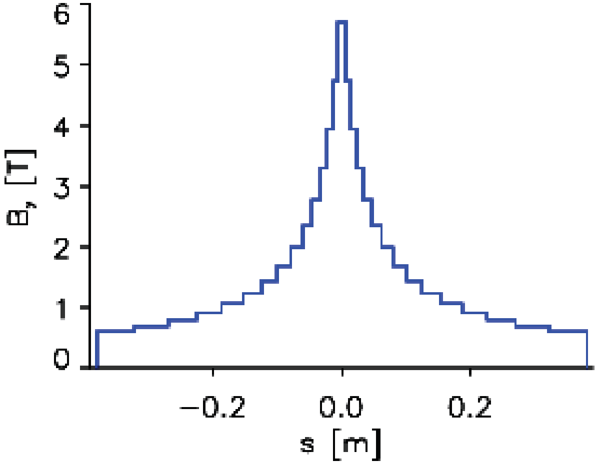
Type of crystal:	protein single crystal	Exposure time:	10 sec
Distance to the xtal:	100 mm	Beam current:	50 mA
Oscillation range:	0.5 deg	ID gap:	10 mm
Wavelength:	1.106 Å	Beam size:	90 x 50 µm

The SLS-2 upgrade project





- Reduction of emittance to 100 pm·rad
  (factor 50 lower compared to SLS-1)
- Miniaturization of vacuum chamber and magnets: 65 mm × 32 mm → ∅ 20 mm
- 7-bend achromat arc using longitudinal gradient bends up to 6 Tesla peak field
- Conceptual design report 2017
  prototype phase 2018-2020
  new storage ring installation 2021-24



Free Electron Laser

Undulator radiation travels with beam, acts like accelerating RF-field.

⇨ **microbunching**

bucket formation at radiation wavelength

⇨ **coherent radiation**

bunch < wavelength → radiates like **one** super-particle. Radiated power:

incoherent $P \sim Ne^2$

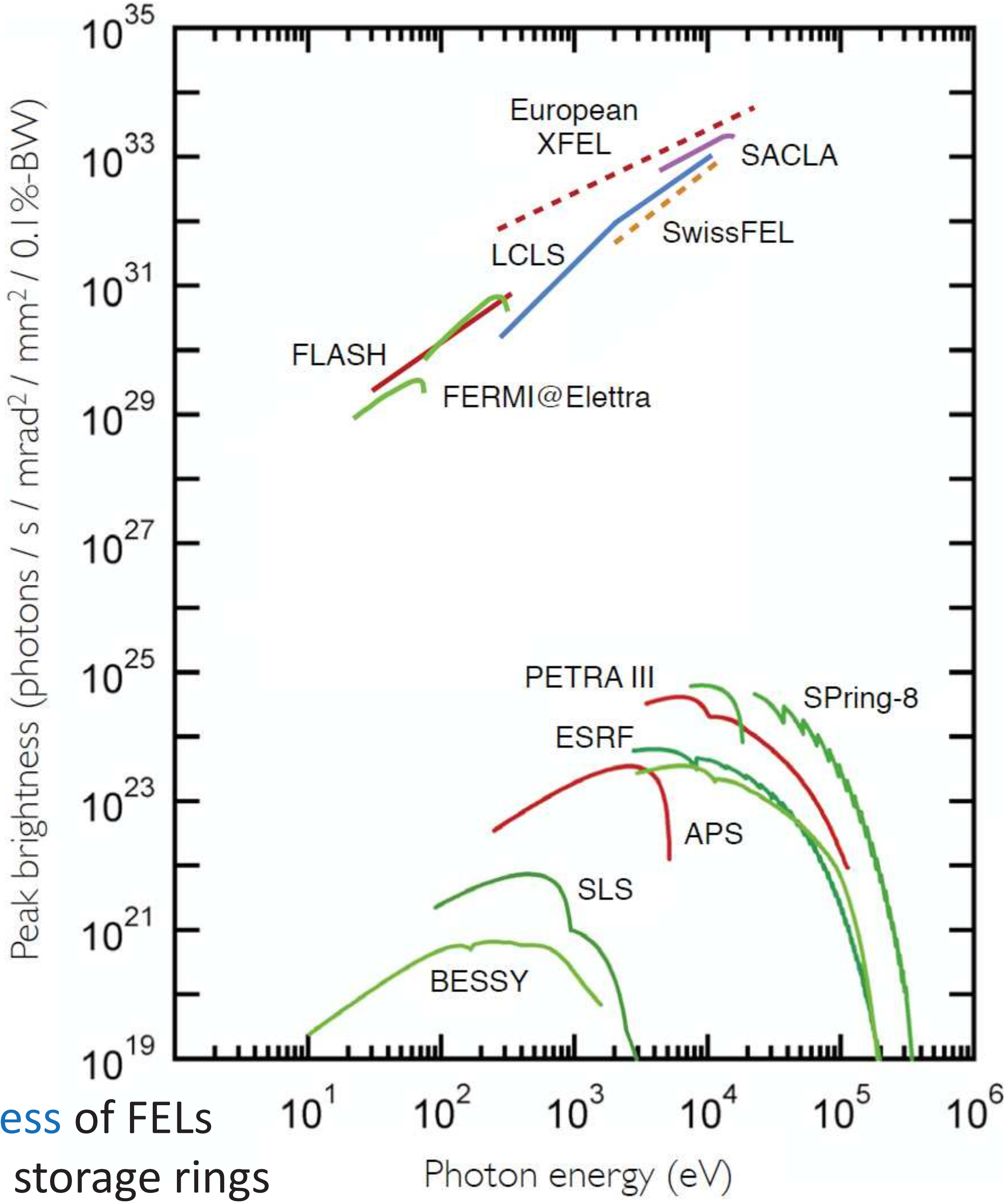
coherent $P \sim (Ne)^2$

⇨ **self-amplification**

exponential increase of power with path length $P \sim e^{s/L_g}$

L_g = gain length

Power saturation at $\approx 22\, L_g$

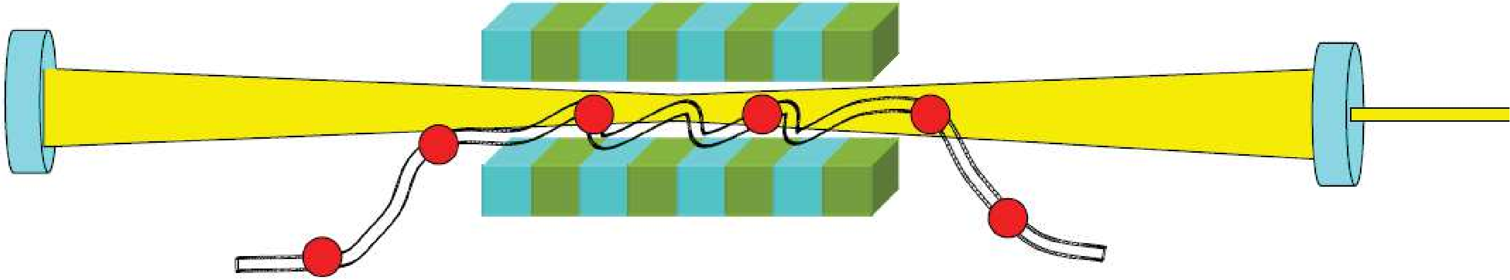


Peak brightness of FELs compared to storage rings

FEL schemes

Oscillator

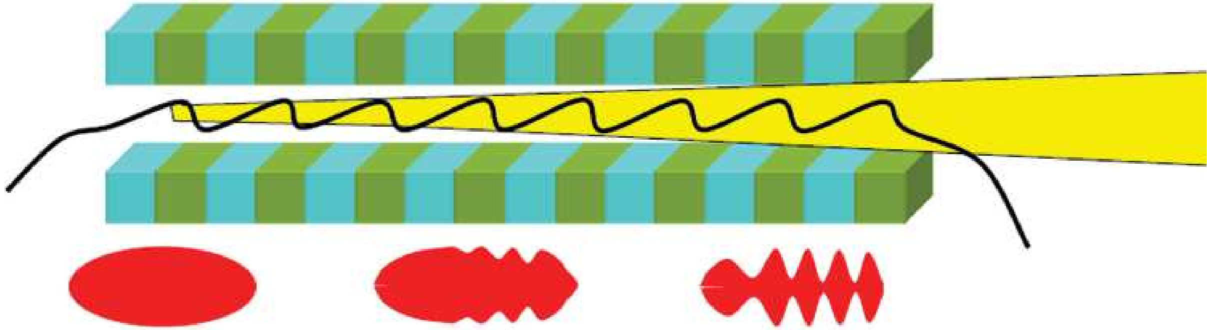
not for X-rays
(no mirrors available)



SASE

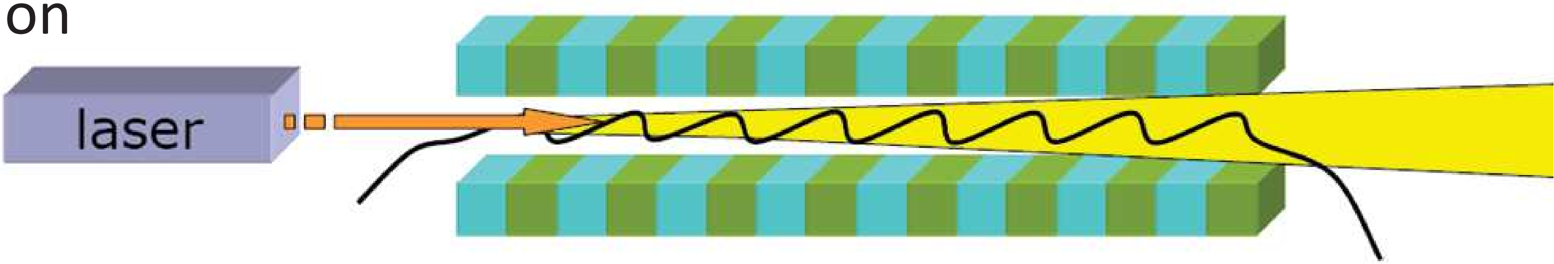
Self **A**mplified **S**pontaneous **E**mission

start-up from noise → unstable



Seeded FEL

microbunching initialization
by external laser



S. Werin, Tutorial on FEL,
http://cas.web.cern.ch/cas/BRUNNEN/Presentations/PDF/tutorial-on-fel-011005.pdf

(X-ray) FEL layout

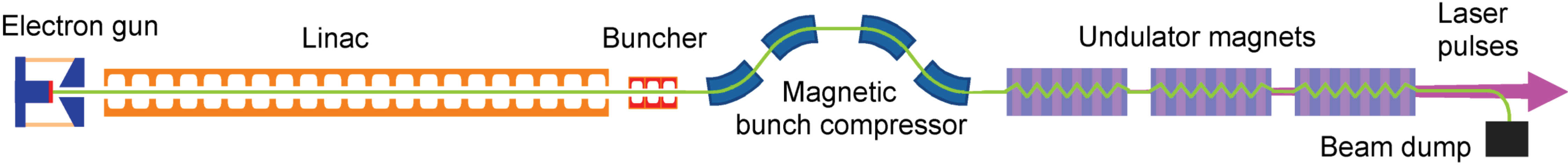
Wavelength $\lambda \sim \frac{\lambda_u}{2\gamma^2}$ $\quad \left.\begin{array}{l} \lambda_u = \text{undulator period } [\sim \mathrm{cm}] \\ \lambda = \text{radiated wavelength } [\sim \mathring{A}] \end{array}\right\} \rightarrow E \approx 2 \ldots 10 \text{ GeV}$

Gain Length $L_g \sim \underbrace{\frac{1}{B_u \sqrt[3]{\lambda_u}}}_{\text{undulator}} \underbrace{\left(\frac{E^4 \epsilon}{\hat{I}}\right)^{\frac{1}{3}}}_{\text{beam}} \rightarrow$ high peak current $\hat{I} > 1$ kA! $\sigma_s < 1$ mm

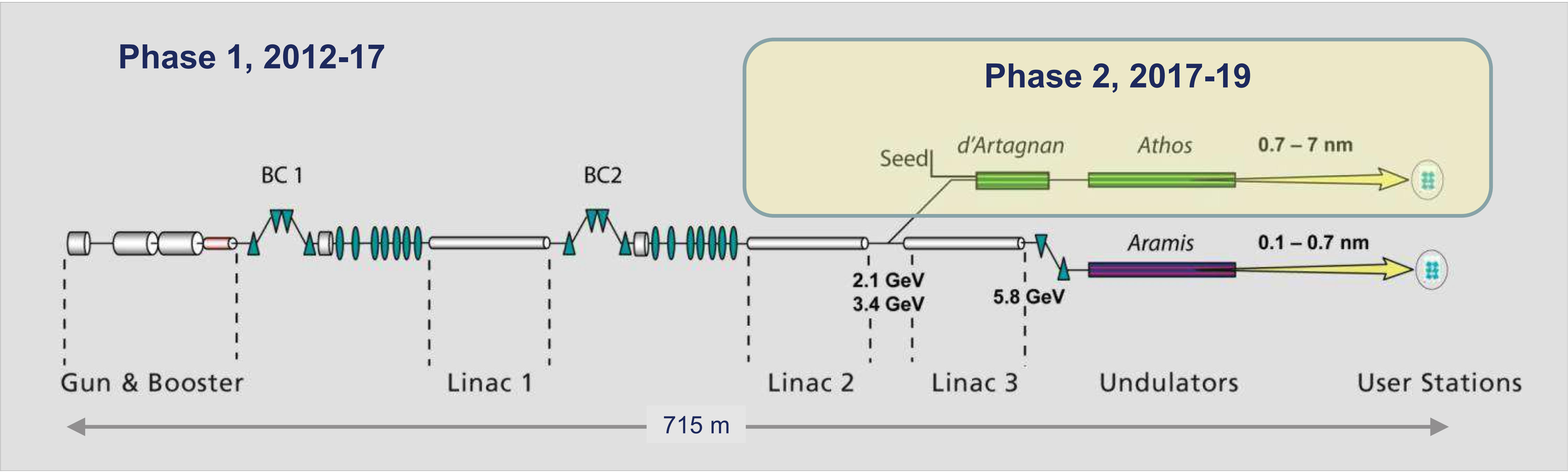
Diffraction limit: $\epsilon < \frac{\lambda}{4\pi} \quad \longrightarrow \quad \epsilon \sim 10^{-11}$ rad m

σ_s, ϵ_x out of reach for storage rings $\longrightarrow$ use linac: $\epsilon \sim \frac{1}{E}$ by pseudo-damping

→ Low emittance electron source developments (laser RF, nano field emitter etc.)



SwissFEL at PSI



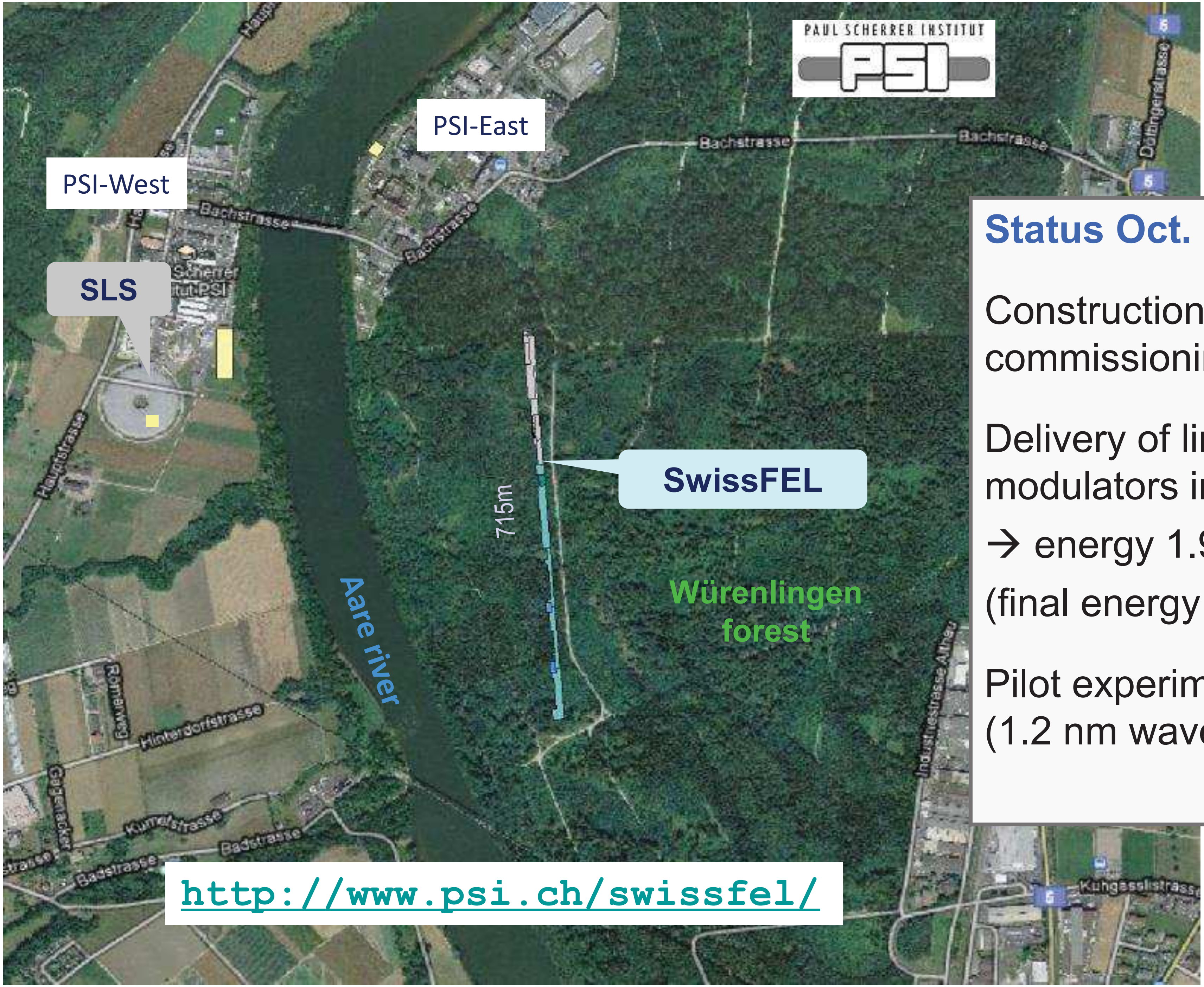
Aramis: 1-7 Å hard X-ray SASE FEL.
Pilot experiments since mid 2017. User operation from end 2018.

Athos : 7-70 Å soft X-ray FEL for SASE & seeded operation .
(2nd phase) User operation 2020.

Beam parameters:

$E_{max} = 5.8$ GeV $\varepsilon_n = 0.35$ µm $\Delta E = 350$ keV

$Q = 10..200$ pC $I_{peak} = 3$ kA 100 Hz rep.rate



http://www.psi.ch/swissfel/

Status Oct. 2017

Construction and commissioning going on

Delivery of linac modulators in progress:

→ energy 1.9 GeV

(final energy 5.8 GeV)

Pilot experiments at 1 keV (1.2 nm wavelength)