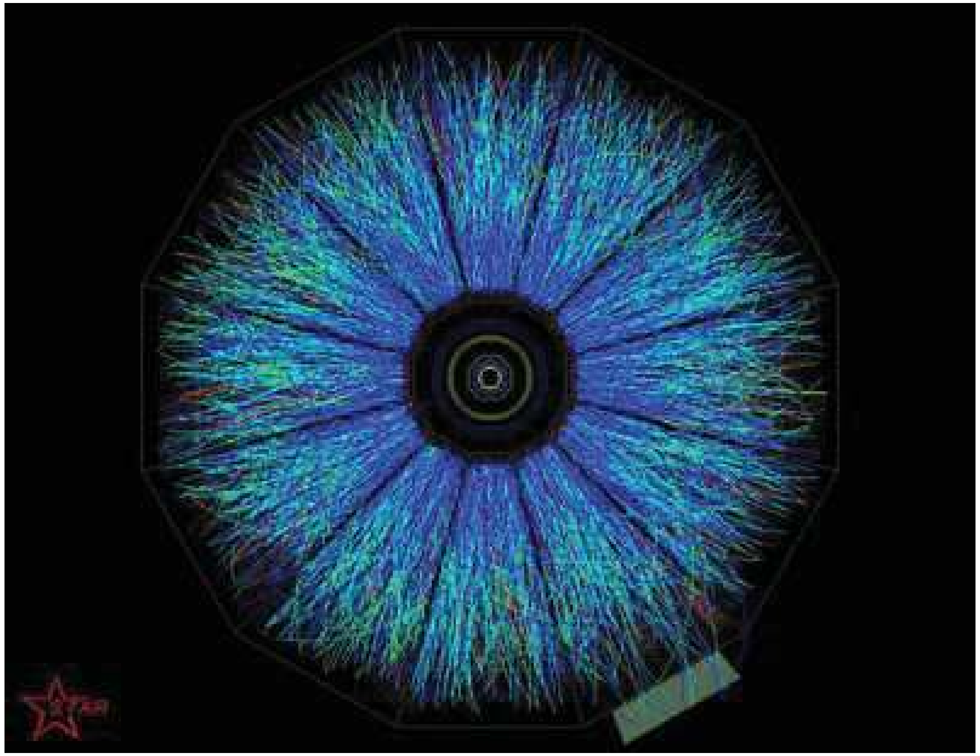
# 7. Luminosity

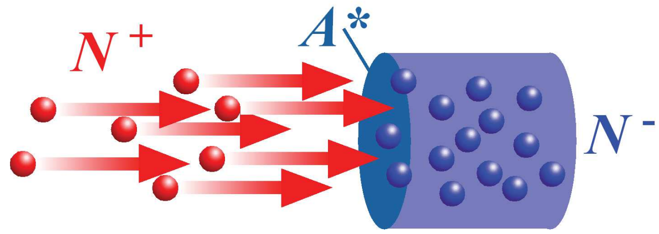
▷ **Luminosity**

Gaussian beams. Hourglass effect. Space charge limit.
Luminosity optimization. Tune spread. Beam-beam limit.
Beam separation. Crab crossing. Crab sextupoles. Beam disruption



# Luminosity with Gaussian beams

Luminosity = particles/time × particle/area




$\mathcal{L} = \frac{N^+}{T} \times \frac{N^-}{A^*}$

($A^*$ common interaction area)

Luminosity = 4-d overlap of particle distributions $\varrho^\pm$ $(\beta = 1)$:

$$\mathcal{L} = f_c \int\int\int\int_{-\infty}^{+\infty} \varrho^+(x, y, s + ct)\, \varrho^-(x, y, s - ct)\, 2cdt\, ds\, dx\, dy$$

$2c$ = relative velocity of bunches in laboratory system,
$f_c = c/b$ = collison frequency, $b$ distance between successive bunches.

Gaussian distributions $\varrho^\pm$, also include [horizontal] crossing angle $2\theta \ll 1$:

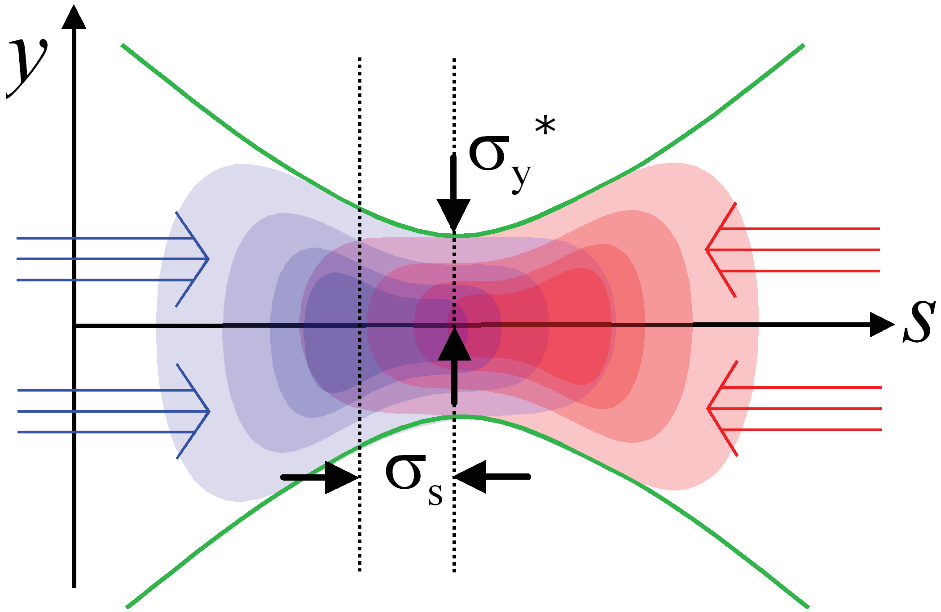
$$\varrho^\pm(x, y, s \pm ct) = \frac{N^\pm}{(2\pi)^{3/2}\sigma_x(s)\sigma_y(s)\sigma_s} e^{-\frac{(x\pm s\theta)^2}{2\sigma_x(s)^2} - \frac{y^2}{2\sigma_y(s)^2} - \frac{(s\pm ct)^2}{2\sigma_s^2}}.$$

Focus at collision point (*) : $\sigma_u(s) = \sigma_u^* \sqrt{1 + \left(\frac{s}{\beta_u^*}\right)^2}, \quad u = x; y$

$$\int \Longrightarrow \qquad \mathcal{L} = \frac{f_c N^+ N^-}{4\pi\, \sigma_x^* \sigma_y^*} \cdot S \quad \longrightarrow \quad A^* = 4\pi\sigma_x^*\sigma_y^* \quad \text{for Gaussian beams}$$

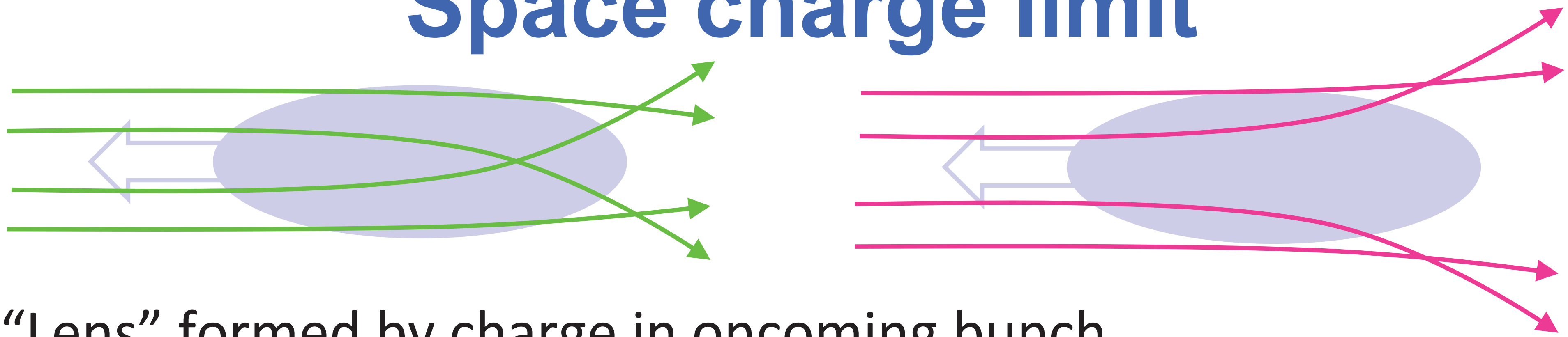
Luminosity suppression factor
*hourglass effect*

$$S = \frac{2}{\sqrt{\pi}\,\sigma_s} \int_0^\infty \frac{e^{-\left(\frac{s}{\sigma_s}\right)^2} e^{-\left(\frac{\theta s}{\sigma_x(s)}\right)^2}}{\sqrt{1 + \left(\frac{s}{\beta_x^*}\right)^2}\sqrt{1 + \left(\frac{s}{\beta_y^*}\right)^2}}\, ds$$




Limit on focus: $\beta^* > \sigma_s \quad \longrightarrow \quad S \approx 0.8 \ldots 0.95$

# Space charge limit



"Lens" formed by charge in oncoming bunch

- very strong: focal length $f \sim \mathrm{cm}$ !
- focusing (e.g. $e^+e^-$) or defocusing (e.g. $pp$)
- non-linear
  - strong for core particles ($x$, $y$ small)
  - weak for halo particles ($x$, $y$ large)
  - even stronger for some $|\Delta s| > 0$ particles

Storage ring collider: betatron tunes $Q_x$, $Q_y$

➔ Beam collision: tune shifts $\Delta Q_{x,y}\,(x, y, \Delta s)$ ⇨ tune spread

➔ Beam-beam tune shift parameter or space charge parameter:
tune shift of center particle $\zeta_{x,y} = \Delta Q_{x,y}\,(0,0,0)$

# Beam-beam limit

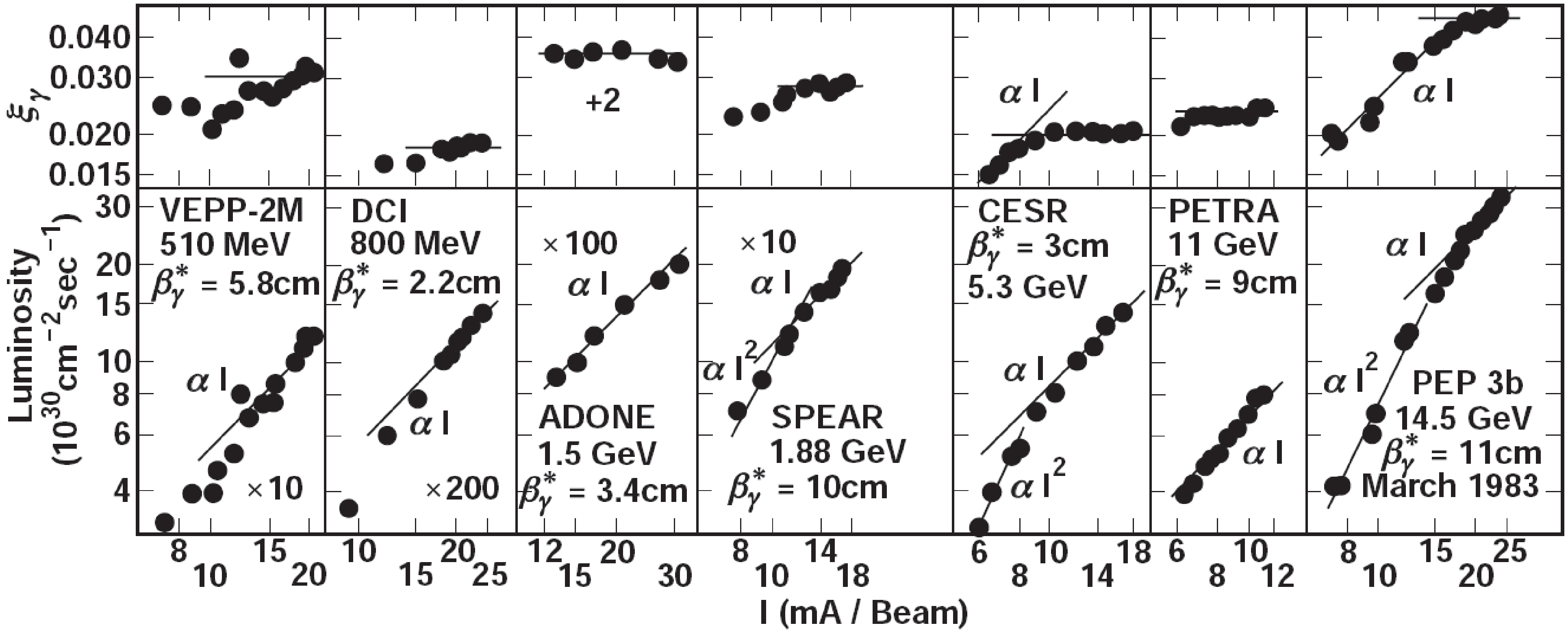
Empirical limits for space charge parameter

$\zeta < 0.05$ in $e^+e^-$ $\quad \zeta < 0.005$ in $p\bar{p}$ collisions

→ Saturation of $\zeta$ with current

→ blow-up of beams, increase of interaction area

→ only linear increase of luminosity: $\mathcal{L} \sim N$, not $\mathcal{L} \sim N^2$




R. Talman, Specific luminosity limits of $e^+e^-$ colliders, PRSTAB 5, 081881 (2002)

# Tune spread

Non-linearity of beam-beam lens $\Longrightarrow$ Tune spread (*beam's footprint*)

Force vs. position:

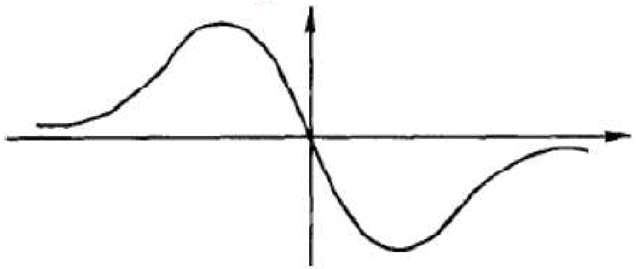
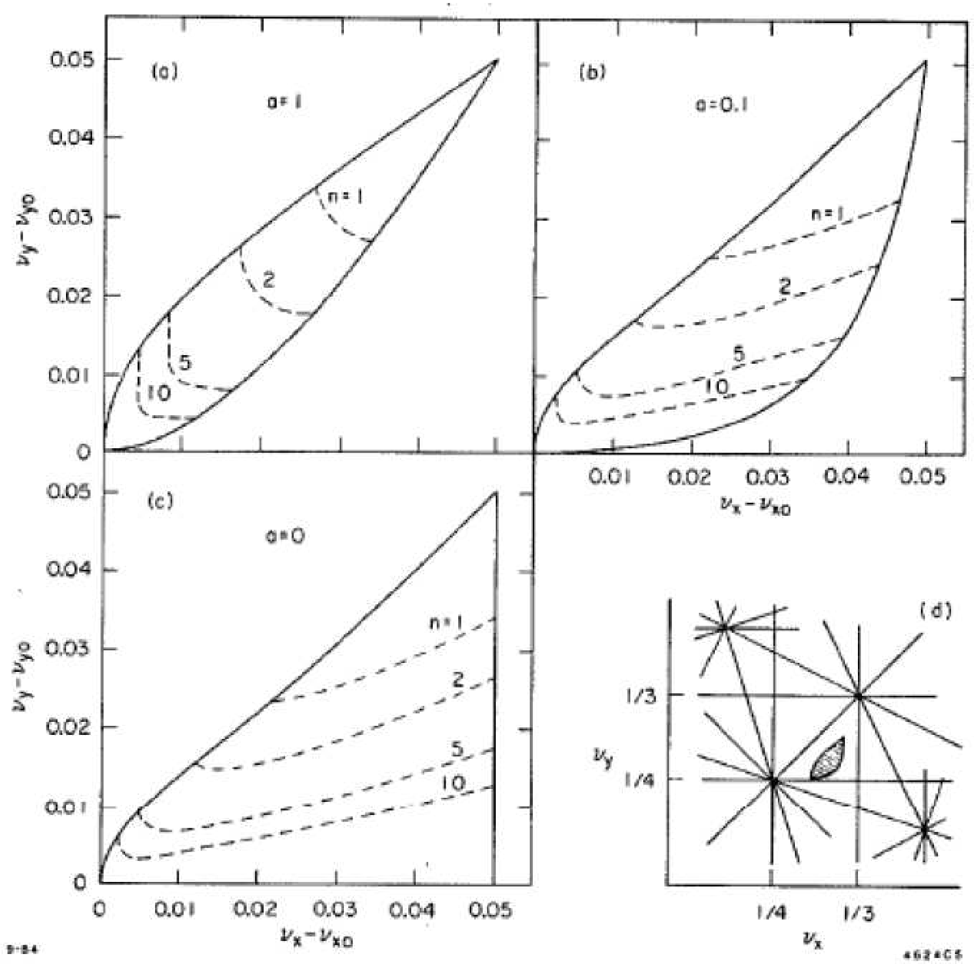



Fig. 11. Beam-beam tune spreads. We assume the two beams have opposite charges. $(\nu_{x0}, \nu_{y0})$ is the unperturbed working point. With beam-beam collisions, the working point extends into a working area. The dotted lines are the contours for particles with amplitudes satisfying $x^2/\sigma_x^2 + y^2/\sigma_y^2 = n^2$. We assume $\xi_x = \xi_y = 0.05$. Case (a) is when the aspect ratio is $a = 1$, i.e. a round beam. Case (b) is when $a = 0.1$, i.e. a flat beam. Case (c) gives the result in the limit $a = 0$. (d) shows fitting the working area (shaded region) into a resonance free region in the tune space.




Ref.: W.T.Weng, *Space charge effects, tune shifts and resonances*, SLAC-PUB-4058, Aug.1986

# ( Traditional ) optimization of luminosity

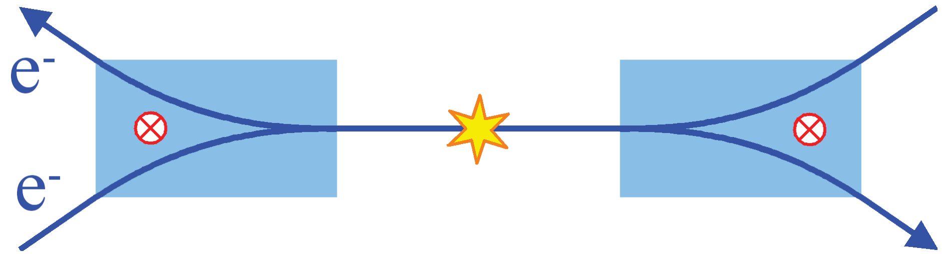
- collision head-on or very small crossing angle
- space parameters $\zeta_{x,y}$ treated as constant (i.e. set to saturation limit)
  - $\zeta_{x,y}$ given by bunch charge and geometry
  - → insert in luminosity formula:

$$\varsigma_u \propto \frac{N\beta_u^*}{\sigma_u^*(\sigma_x^* + \sigma_y^*)}; \quad u = x, y$$

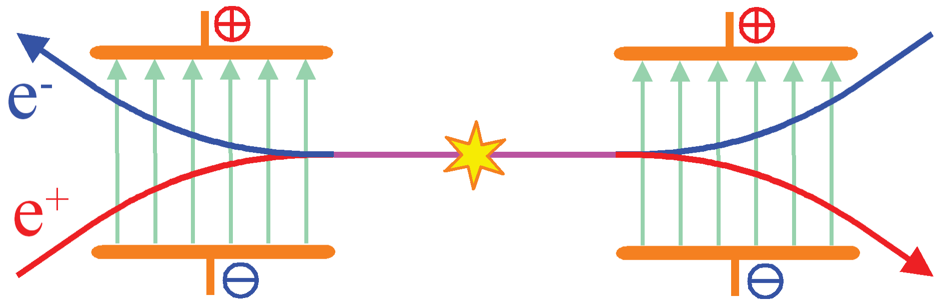
Luminosity (flat beams) $\mathcal{L} \sim E^2 \frac{\varepsilon_x}{b\beta_y^*} \varsigma_x \varsigma_y S$

- → large (!) emittance
- → sharp focus – *but*
  - avoid luminosity suppression due to hourglass effect: $\beta^* > \sigma_s$ for $S \approx 1$
  - avoid longitudinal beam-beam effect: energy modulations for large $x'$, $y'$
  - avoid tune shift increase $\Delta Q > \zeta_{x,y}$ for $|\Delta s| > 0$
- → short distance between successive bunches
  - limit: $b = \lambda_{\mathrm{RF}}$ , bunch distance = RF wavelength
  - avoid parasitic crossings

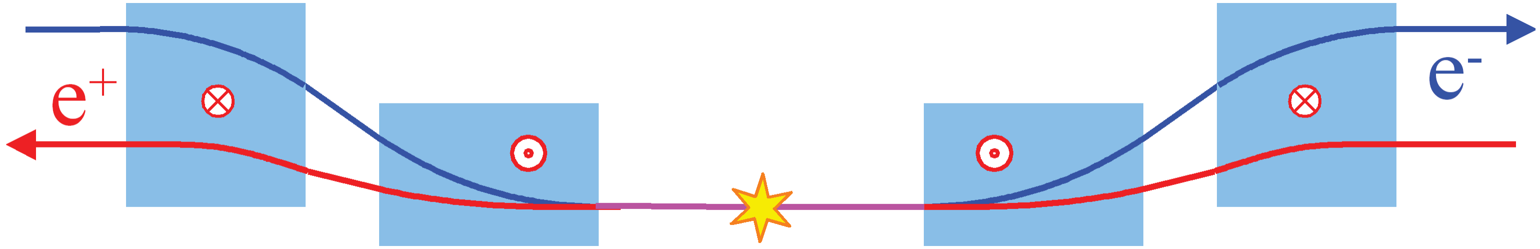
# Beam separation schemes



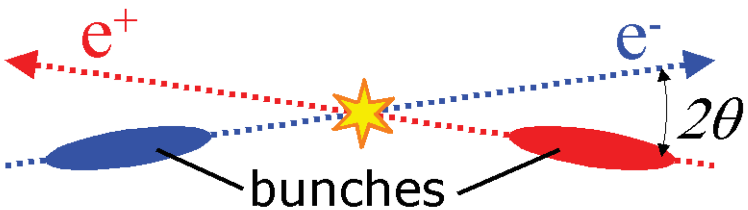

Magnetic separation for head-on collision of identical particles at same energy




Electrostatic separation for head-on collision of particle/antiparticle at same energy




Asymmetric magnetic separation for head-on collision of particle/antiparticle at different energies to boost the center of mass system.



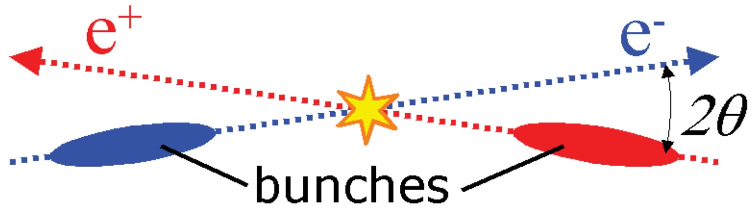

Crossing angle
- high collision frequency
- reduced luminosity due to incomplete overlap

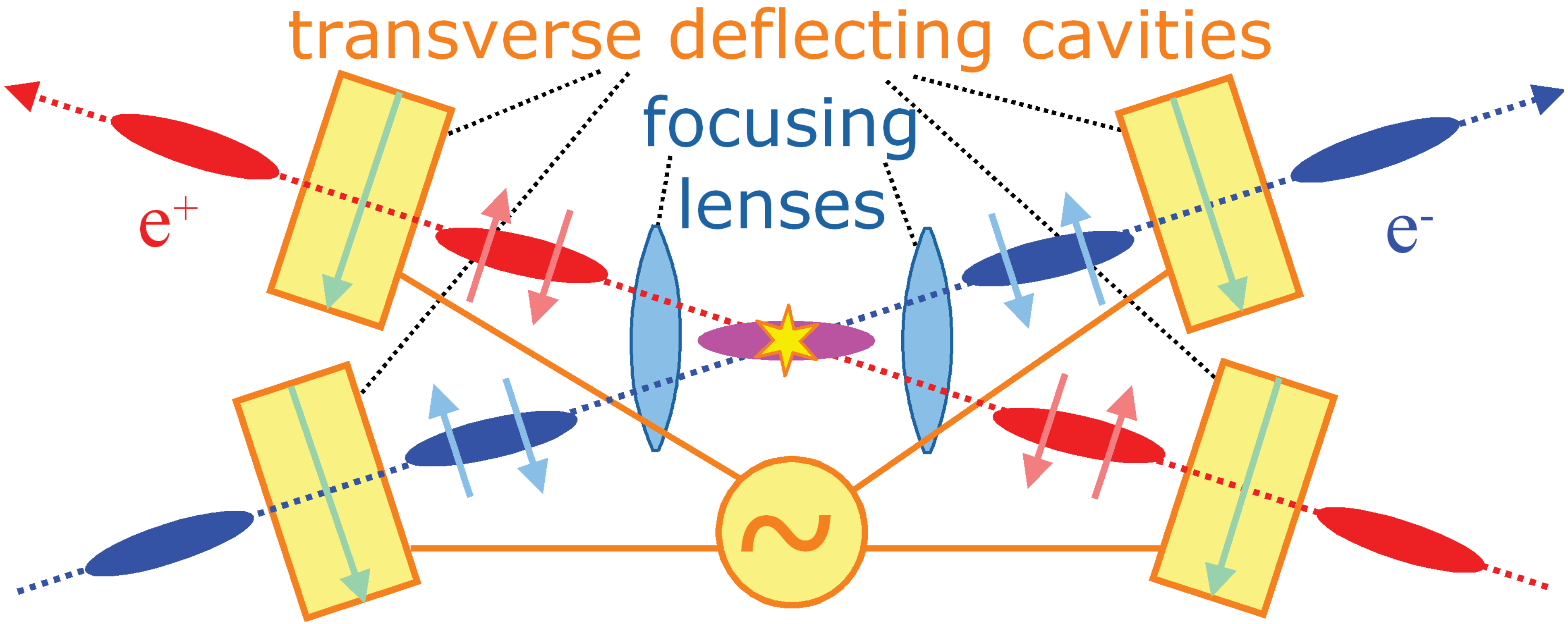
# Crab Crossing

Crossing at angle $\theta << 1$

- ☺ high bunch frequency
- ☹ reduced overlap/luminosity
- ☹ excitation of transverse/longitudinal coupling resonances

⇨ restore head-on collision:



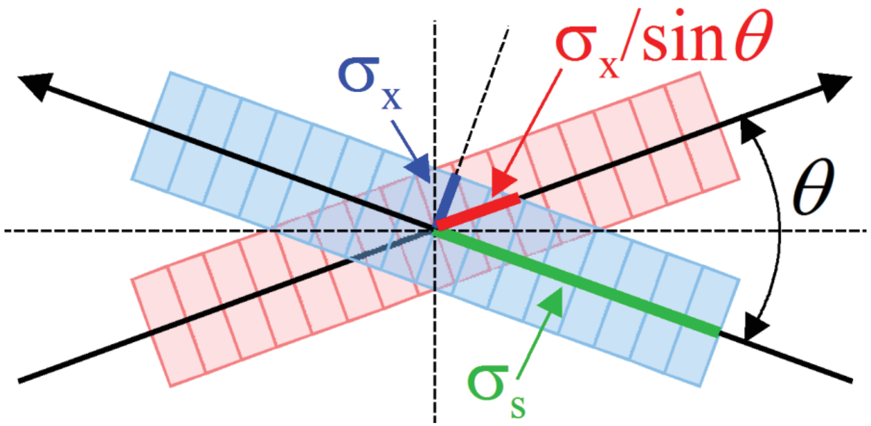




In use at KEK B-factory, reached $\mathcal{L} > 2\cdot 10^{34}\ \mathrm{cm^{-2}s^{-1}}$ ☺

# Crossing collision

for flat beams ( $\sigma_y << \sigma_x << \sigma_z$ )

large crossing angle (some degree)




## 1. Reduction of interaction zone

$2\sigma_s \to L_{cross} = 2\sigma_x / \sin\theta$

⇨ reduction of $\mathcal{L}$ and $\zeta$

## 2. Low emittance beams

⇨ increase $\zeta$ to space charge limit again

⇨ $\mathcal{L}$ increases too ⇨ $\mathcal{L}$ independent of crossing angle $\theta$ !

## 3. Micro-beta focusing

Criterion: $\beta^* > \sigma_s \to \beta^* > L_{cross} << \sigma_s$

⇨ sharper focusing, very small $\beta^*$

⇨ increase of luminosity: $\mathcal{L} \sim 1/\beta^*$

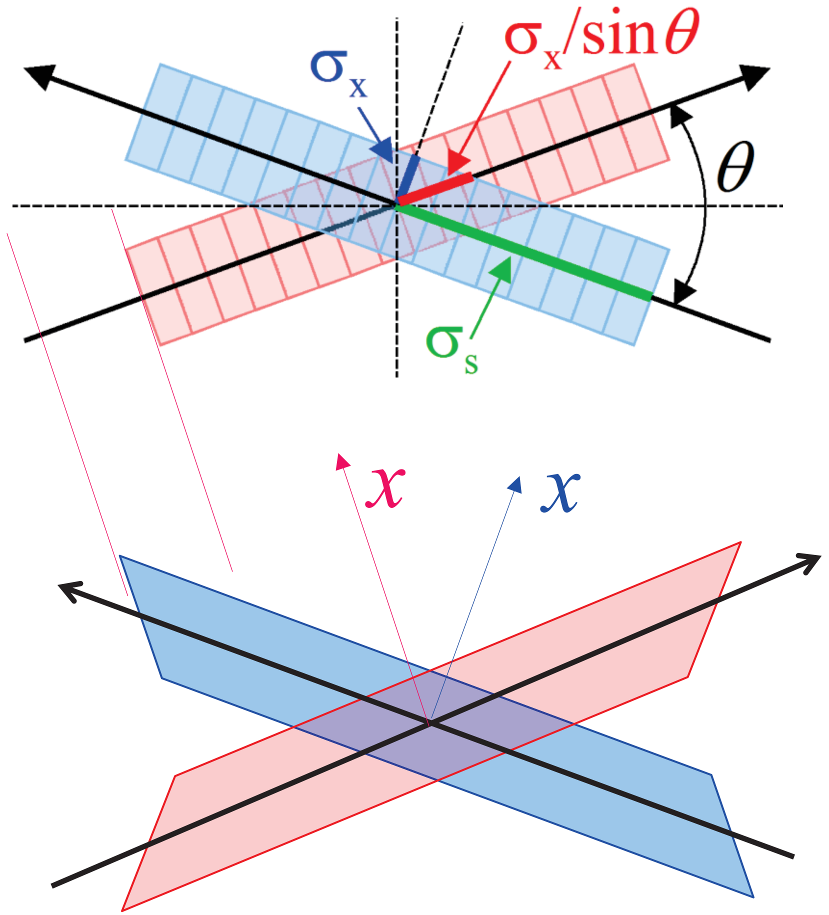
⇨ decrease of space charge parameter: $\zeta \sim \beta^*$

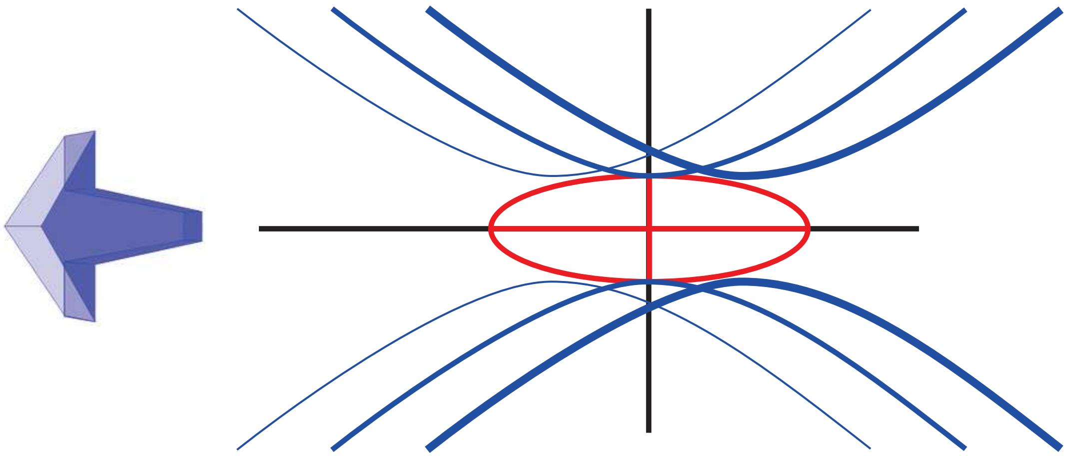
D. N. Shatilov & M. Zobov, ICFA BD NL 37 (2005) 99-109

P. Raimondi, D. N. Shatilov & M. Zobov, EPAC-08, Genoa, 2008, WEPP045

# 4. Crab waist

Lateral shift of focus during collisions
⇨ transverse-longitudinal coupling forces
⇨ beam instability!



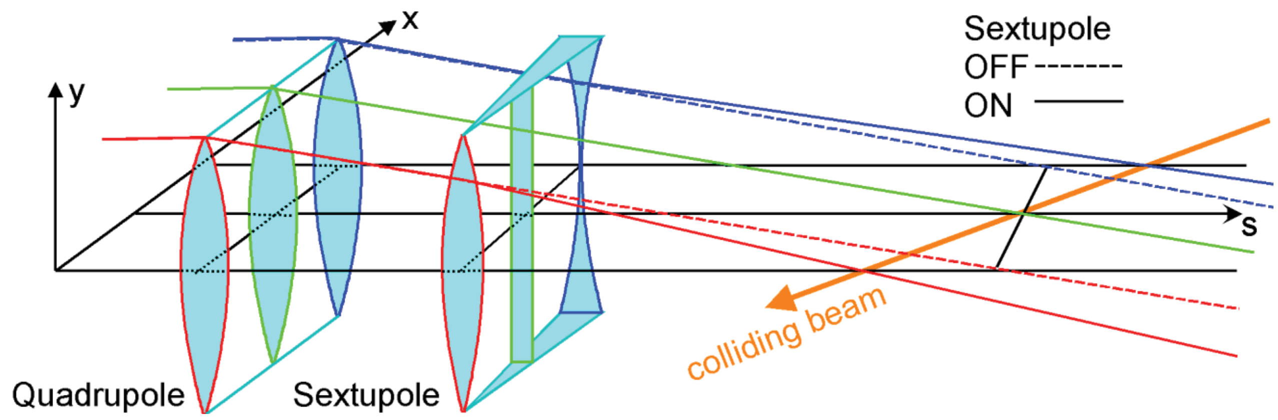



colliding "blue beam" as seen from "red beam"

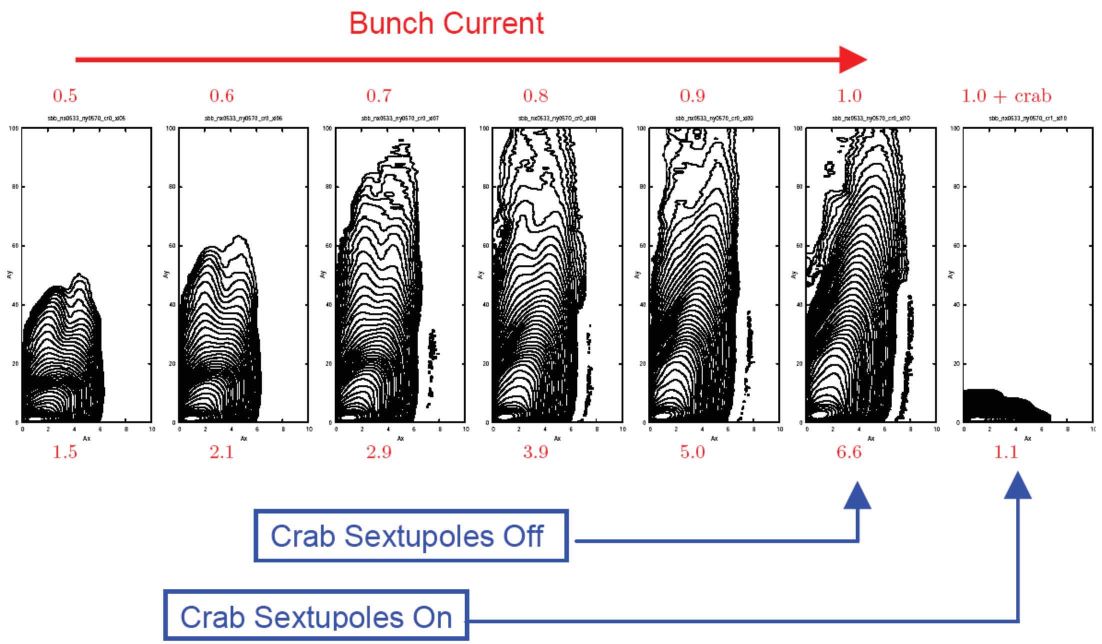
To do: adjust $x$-dependent $s$-position of focus, such that it coincides with the other beam's $x$-axis

⇨ $x$-$s$ shear of bunch ⇨ *How to do?*

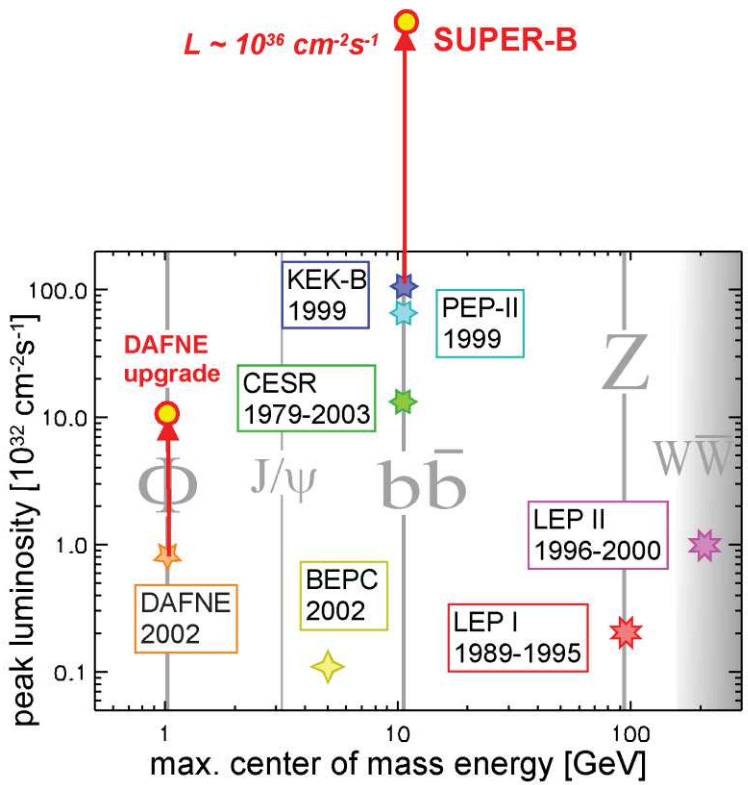
# “Crab sextupole”




Horizontal variation of focal length




Suppression of vertical blow-up (beam instability)




Expected luminosity increase

Crab-crossing successfully tested at DAΦNE

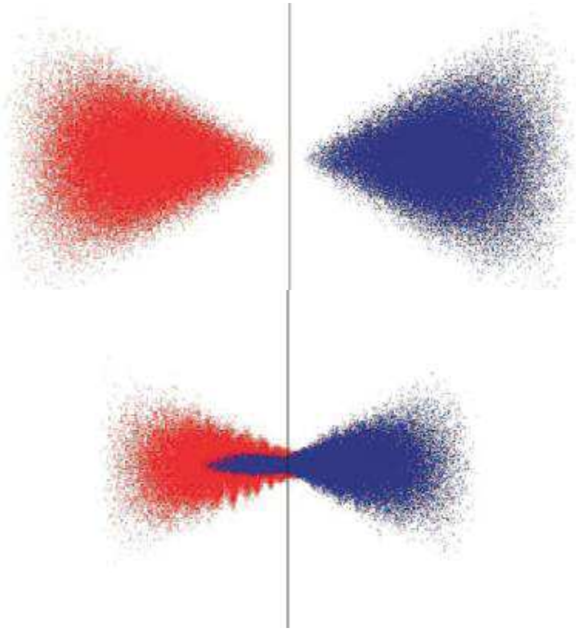
# Beam disruption

Linear collider: no re-use of beam ⇨ no limit on $\zeta$-parameter

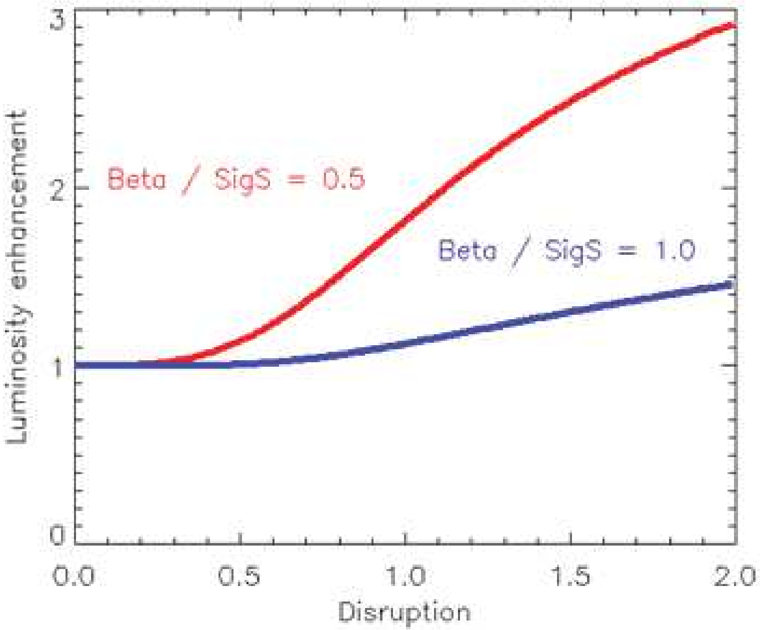
⇨ beam-beam lens: focal length < bunch length, $f < \sigma_s$

⇨ self-focussing (pinch-effect) ⇨ luminosity enhancement

⇨ beam disruption, parameter $d_u = \frac{\sigma_s}{f} = \frac{2r_e}{\gamma} \frac{N\sigma_s}{\sigma_u(\sigma_x + \sigma_y)} = -4\pi\varsigma_u \frac{\sigma_s}{\beta_u} \quad u = x; y$



📄 M. Boronina et al., PAC-07, Albuquerque 2007, THPAN060




Luminosity enhancement

## Linear collider luminosity limitations

- sub-$\mu$m alignment of final focus
- beamstrahlung = synchrotron radiation in field of oncoming bunch

⇨ photon recoil ⇨ undefined center of mass energy

⇨ direct gamma-background

⇨ pair production background