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Minimum emittance superbend lattices ?

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Minimum emittance

The minimum emittance obtainable from a *homogenous* (i.e. constant longitudinal field) bending magnet with full deflection angle Φ and full length L is given by

$$\epsilon_{\mathrm{TME}}[\text{nm·rad}] = K \frac{\Phi^3}{12\sqrt{15}}, \quad \text{with} \quad K := 1470\, \frac{(E[\text{GeV}])^2}{J_x} \tag{1}$$

if horizontal betafunction and dispersion in the magnet's center are adjusted according to the constraints

$$\beta_{xo} = \frac{L}{2\sqrt{15}} \qquad \eta_o = \frac{h\, L^2}{24} \tag{2}$$

Here $h = 1/\rho = \Phi/L$ is the magnet's curvature.

As first suggested by A. Wrulich in 1992, later elaborated analytically by R. Nagaoka [2, 3] and numerically by J. Guo and T. Raubenheimer [1], a bending magnet with longitudinal field variation may provide an emittance significantly below the minimum emittance of a homogeneous bending magnet:

The equilibrium emittance in a flat (i.e. no vertical bends) storage ring is given by

$$\epsilon = K \frac{I_5}{I_2} \tag{3}$$

with the synchrotron integrals

$$I_2 = \int h(s)^2\, ds \quad \text{and} \quad I_5 = \int h(s)^3\, \mathcal{H}(s)\, ds$$

where $h(s)$ is the local curvature of the magnet and

$$\mathcal{H}(s) = \gamma_x(s)\eta(s)^2 + 2\alpha_x(s)\eta(s)\eta'(s) + \beta_x(s)\eta'(s)^2 \tag{4}$$

With a beam focus in the magnet's centre, $\mathcal{H}$ will have a minimum value there and grow towards the magnet's edges. Consequently, increasing the curvature $h(s) = B(s)/(B\rho)$ in the dipole centre and lowering it in the outer regions while keeping the integral

$$\int_L h(s) ds = \Phi \tag{5}$$

constant, will compensate for the variation of $\mathcal{H}$ and lead to a lower emittance. Finding a function $h(s)$ for minimizing eq.3 under the constraint of req.5 represents an isoperimetric variational problem, which however can not be solved in general

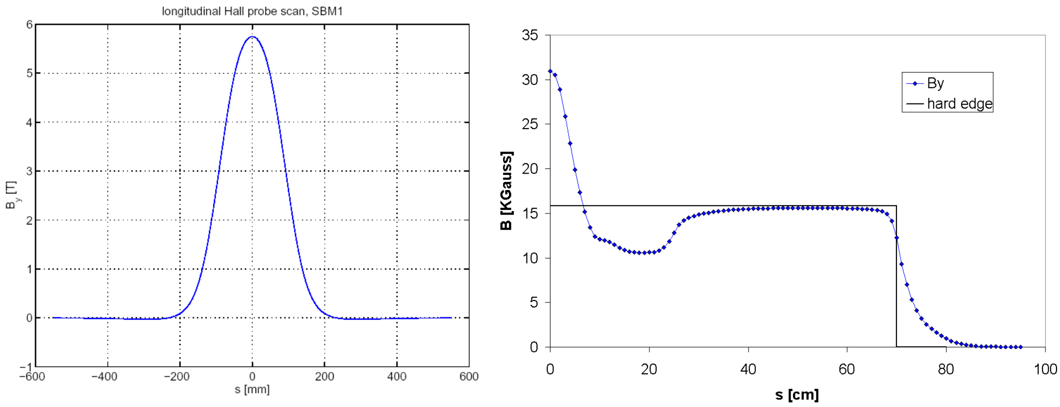


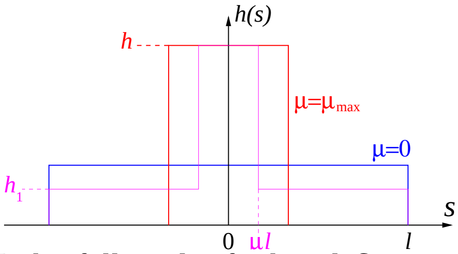
Figure 1: Superbends as installed in ALS (left, taken from [4]) and in SLS (right)

analytically. Instead numerical minimizations [1] or analytical minimizations assuming special functions for $h(s)$ [2, 3] were done. The ideal functions to be found correspond to technically non-feasible magnets of infinite length with infinite curvature in the centre. Here we will try a simpler approach which includes realistic boundary conditions and gives an analytic solution.

Superbends

Real magnets with increased central field, called "superbends" are installed in several medium light sources for production of hard X-rays. ALS runs superconducting magnets with 5.6 Tesla peak field [4], SLS plans for normalconducting 3.2 Tesla magnets (March 2005), see fig.1.

A superbend can be simplified as consisting from two components: a central high field part with given curvature h, corresponding to the highest possible field, and an outer low field part with lower curvature h_1.



With $l = L/2$ the *half* length and Φ the full angle, further defining μ as a measure how far the high field region extends, eq.5 gives

$$h\mu l + h_1(1-\mu)l = \frac{\Phi}{2} \qquad \longrightarrow \qquad \mu_{\max} = \frac{\Phi}{2hl}$$

The emittance of this bending magnet is given by

$$\varepsilon = K \frac{I_{50} + I_{51}}{I_{20} + I_{21}}$$

where indices "0" and "1" indicate contributions from the central, resp. the outer part.

For the propagation of optical parameters in the magnets a simplified sector magnet matrix is used by assuming that $\Phi \ll 1$, which is an acceptable approximation for most light sources. Then the 3×3 horizontal, dispersive matrix of a dipole of length l and curvature h becomes:

$$\begin{pmatrix} 1 & s & hs^2/2 \\ 0 & 1 & hs \\ 0 & 0 & 1 \end{pmatrix}$$

This matrix determines the propagation of the $\mathcal{H}$-function and allows to calculate the I_5 integrals:

$$\varepsilon(\beta_o, \eta_o, \mu) = K \frac{h^3 \int_0^{\mu l} \mathcal{H}_o(s)\, ds \,+\, h_1^3 \int_{\mu l}^{l} \mathcal{H}_1(s)\, ds}{h^2 \mu l + h_1^2 (1-\mu) l}$$

β_o, η_o are the horizontal optical parameters in the magnet centre (the index “x” has been dropped), which we want to calculate analogous to eq.2. Due to symmetry we set $\alpha_o = \eta' = 0$.

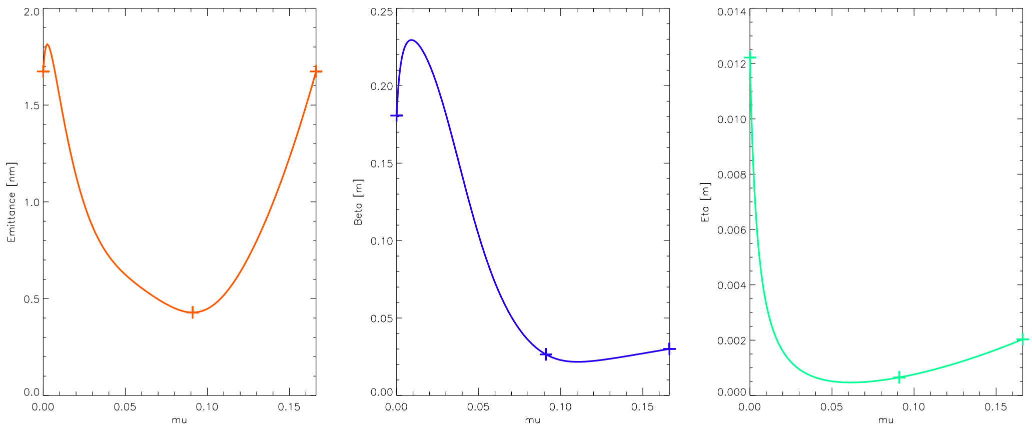
The analytic solution of the integral above is rather lengthy and best handled by an algebraic code like MATHEMATICA. The matching conditions for the initial parameters are obtained by

$$\frac{\partial \varepsilon\,(\beta_o, \eta_o, \mu)}{\partial \eta_o} \stackrel{!}{=} 0 \rightarrow \eta_o\,(\beta_o, \mu) \quad \Longrightarrow \quad \frac{\partial \varepsilon\,(\beta_o, \mu)}{\partial \beta_o} \stackrel{!}{=} 0 \rightarrow \beta_o\,(\mu) \quad \Longrightarrow \quad \varepsilon\,(\mu)$$

The results are backsubstituted to get the minimum emittance as a function of the μ-parameter.

Example

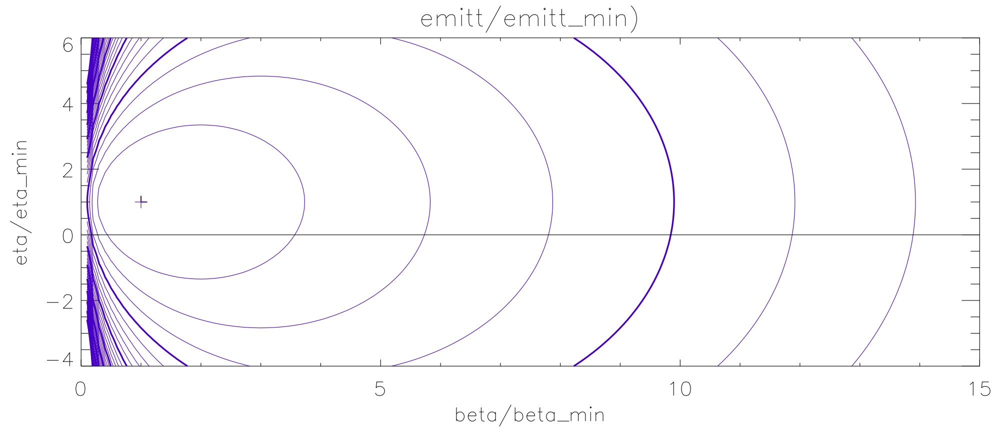
With a beam energy of 2.4 GeV and a maximum field of 7.2 T (assuming a superconducting coil), the bend angle was set to 12°, and the magnet length was limited to 1.4 m. Emittance, central beta and dispersion as functions of μ are shown in this figure:



The μ-range extends from 0 to $\mu_{\max}$, corresponding to a pure normalconducting magnet of full length $2l$, resp. to a pure superconducting magnet of length $2\mu_{\max}l$. Both these limiting cases result in the same emittance, since according to eq.1, the minimum emittance of a homogenous magnet depends only on the bend angle. The μ-value providing the lowest emittance was calculated numerically, since an analytical solution was not possible and is marked in the plot above. For this example, it is almost four times lower than the minimum emittance from eq.1!

A cell based on this 12° superbend provides an emittance of 0.43 nm rad at 2.4 GeV and is shown in fig.2. Although the chromaticity is extreme ($\xi/\nu \approx -10!$), correction with sextupoles integrated in the quadrupoles works well and provides dynamic apertures which are not hopeless. Nevertheless, the example is academic: the extreme optics would be very sensitive to all kind of errors. Further, the rms energy spread amounts to $1.9{\cdot}10^{-3}$, since it scales with $I_3 = \int |h(s)|^3\, ds$ and thus becomes large in the presence of superbends. Therefore, the large dispersion in the straights would determine the source size.

As for the homogenous magnet [5], it turns out, that the emittance does not grow very much if the center beta and eta deviate from the ideal conditions. The contours in the plot below indicate equal emittances as functions of deviations from the ideal condition marked as $+$. The first contour gives double emittance, the thick contour 5 times the value:



Even increasing the central beta and dispersion both by a factor of 4 gives only the double emittance, i.e. still below 1 nm. All light sources run at emittances which are approx. $3\ldots5$ larger than the minimum theoretically possible for their magnets. In case of the superbend lattice the factor 4 gain in the theoretical minimum thus may help to end up in an emittance range comparable to the theoretical minimum of existing light sources.

A more realistic example requires dispersion suppression for the straights, since the energy spread will be large due to superbending fields, and slight relaxation of the optics for the sake of feasibility and robustness.

The minimum emittance from dispersion suppressor magnets is 3 times the minimum emittance from eq.1 [5]. The optimum angle of the dispersion suppressor

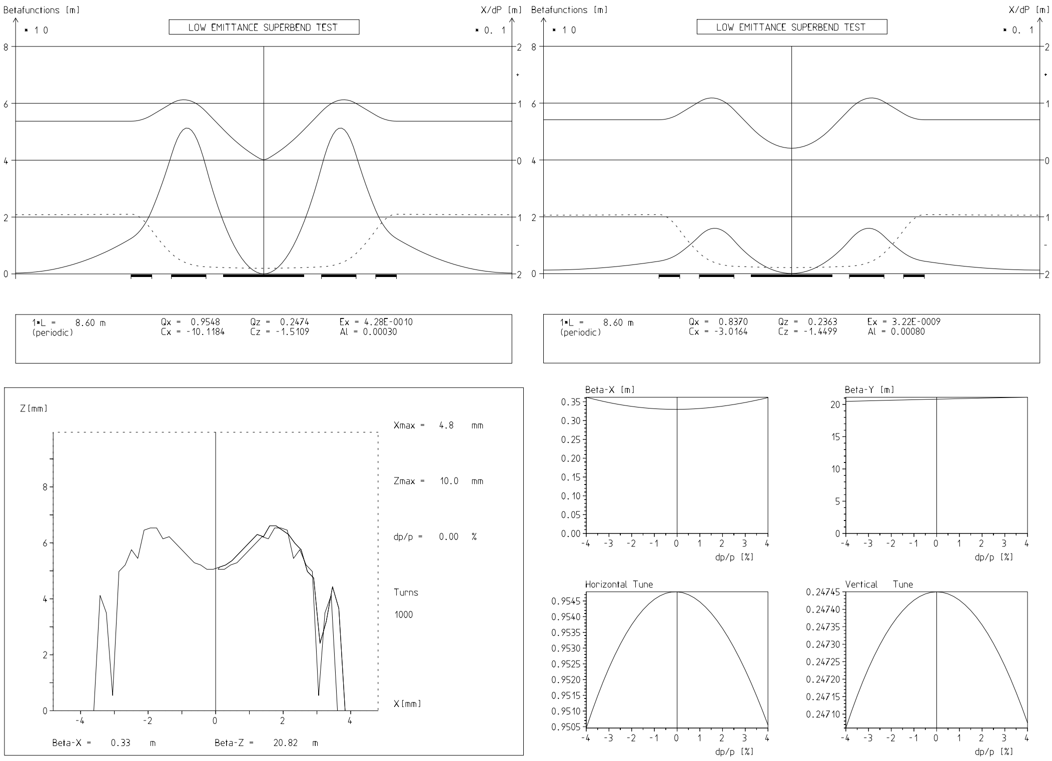


Figure 2: Example of a superbend minimum emittance cell

The cell is based on a 12° bending magnet of 1.4 m length with a superconducting central region of length 0.13 m and a field of 7.2 T. The outer field is 0.6 T. The optimum optics fulfilling constraints on central beta and dispersion is shown in the upper left, it provides 0.43 nm emittance at 2.4 GeV. Two combined quadrupole/sextupole families do the focusing and the chromaticity correction. The dynamic aperture is shown in the lower left, it amounts to approx. 27 mm mrad in the horizontal (sufficient for injection) and 1 mm mrad in the vertical (compatible with mini gap insertions). Second order chromaticity is small and energy acceptance large as shown in the lower right.

For normal operation, e.g. in case of quenching of the super conducting coil, the main field of the dipole can be raised to 1.2 T. This gives the optics shown in the upper right, with an emittance of 3.2 nm. Also intermediate settings would be feasible in order to tune the superbend spectrum.

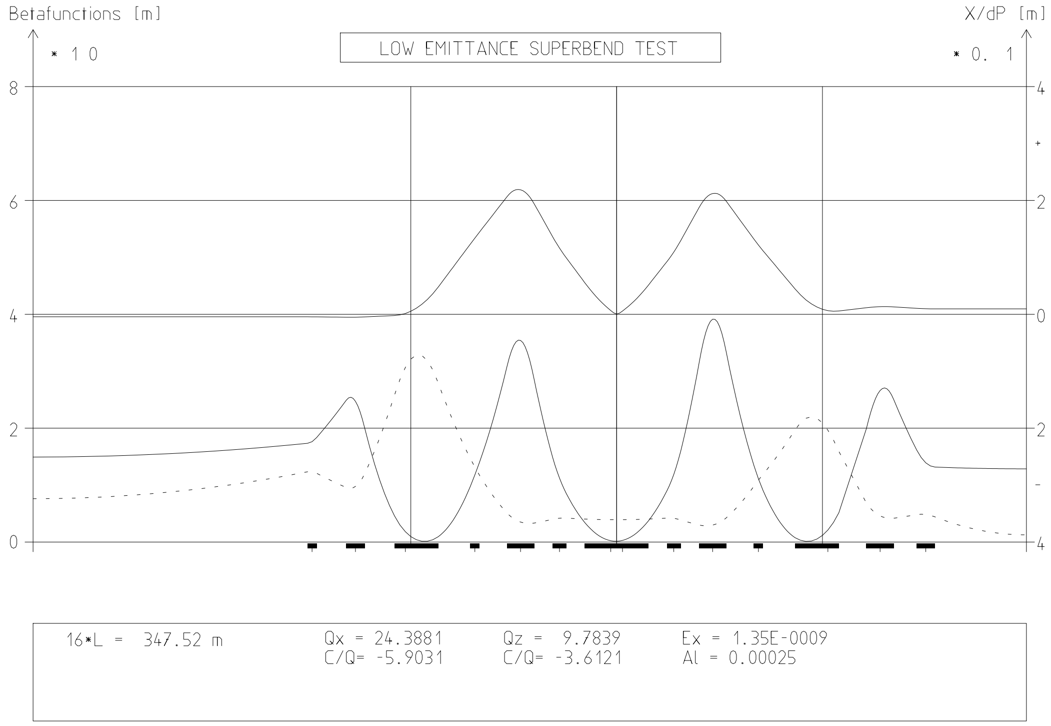


Figure 3: Example for a modified TBA lattice containing central superbends (one sixteenth shown)

(DS) magnet thus has to be chosen such, that it delivers the same emittance as the superbend (SB), i.e.

$$\Phi_{\rm DS} = \Phi_{\rm SB} \times \sqrt[3]{F/3}$$

where $F = \varepsilon/\varepsilon_{\rm TME}$ is the emittance reduction factor for the superbend at optimum μ. For the 12° superbend considered before with $F = 0.26$ we thus obtain 5.28° as dispersion suppressor bending angle. A value of 5.25° is an obvious choice, then 16 modified triple bend achromats make a ring.

A scratch lattice of that type is shown in figure 3: The minimum condition was relaxed, resulting in 1.35 nm at 2.4 GeV for a ring of 350 m circumference offering 16×4 m and 16×10 m [almost] dispersion free straight sections.

However, more work is required to optimize the dynamic aperture of this lattice.

References

[1] J. Guo and T. Raubenheimer, "Low emittance e^-/e^+ storage ring design using bending magnets with longitudinal gradient", Proc. EPAC 2002, Paris, p.1136

[2] R. Nagaoka, "Attempt to minimize the emittance with longitudinal dipole field variation", talk given at Small Emittance Light Source Workshop, Lund, Mar.12, 2004, download from `http://maxsun5.maxlab.lu.se/maxlab/conference/small-emittance/`

[3] R. Nagaoka, "Emittance Minimization with longitudinal field variation", talk given at ESLS-12, Hamburg, Nov.16, 2004, download from `http://esls.desy.de`

[4] D. Robin et al., "Superbend project at the Advanced Light Source", Proc. PAC 2001, Chicago, p.2632

[5] A. Streun, "Practical guidelines for lattice design", course given at CERN accelerator school, Benodet, Aug.30-Sep.9, 1999, Internal report SLS-TME-TA-1999-0014, download from `http://slsbd.psi.ch/pub/cas/`

Appendix: Minimum emittance, central beta function and dispersion as function of μ-parameter (with central curvature h, magnet half length l and full deflection angle Φ):

$$\begin{aligned}
\varepsilon\,[\mathrm{nm}] \;=\; & \frac{1470\,(E\,[\mathrm{GeV}])^2}{J_x} \times \frac{1}{12\sqrt{15}} \\
& \times \{ \; \left(8\,h^3\,l^3\,\mu\;(-1+2\,\mu) - 12\,h^2\,l^2\,\mu^2\,\Phi + 6\,h\,l\,\mu\,\Phi^2 - \Phi^3\right) \\
& \times (\; 32\,h^5\,l^5\,\mu^3\;(-1+2\,\mu) - 32\,h^4\,l^4\,\mu^4\,\Phi + 8\,h^3\,l^3\,\mu^3\,\Phi^2 \\
& \quad - 4\,h^2\,l^2\,\mu^2\,\Phi^3 + 4\,h\,l\,\mu\,\Phi^4 - \Phi^5\;) \\
& \times (\; 1280\,h^8\,l^8\,\mu^6\;(-1+\mu\;(-1+5\,\mu)) \\
& \quad - 640\,h^7\,l^7\,\mu^5\;(-9+\mu\;(7+13\,\mu\;(1+\mu)))\;\Phi \\
& \quad + 640\,h^6\,l^6\,\mu^4\;(-9+\mu\;(3+\mu\;(25+\mu\;(19+4\,\mu))))\;\Phi^2 \\
& \quad - 320\,h^5\,l^5\,\mu^3\;(-9+\mu\;(-1+\mu\;(41+\mu\;(41+12\,\mu))))\;\Phi^3 \\
& \quad + 80\,h^4\,l^4\,\mu^2\;(-9+\mu\;(-5+\mu\;(77+\mu\;(107+40\,\mu))))\;\Phi^4 \\
& \quad - 8\,h^3\,l^3\,\mu\;(-9+\mu\;(-9+\mu\;(245+\mu\;(413+200\,\mu))))\;\Phi^5 \\
& \quad + 112\,h^2\,l^2\,\mu^2\;(4+\mu\;(7+4\,\mu))\;\Phi^6 \\
& \quad - 16\,h\,l\,\mu\;(4+\mu\;(7+4\,\mu))\;\Phi^7 + (4+\mu\;(7+4\,\mu))\;\Phi^8\;)\;\}^{\frac{1}{2}} \\
/ \quad & \{2\;(-1+\mu)\;\left(4\,h^2\,l^2\,\mu - 4\,h\,l\,\mu\,\Phi + \Phi^2\right) \\
& \times \left(8\,h^3\,l^3\,\mu\;(-1+2\,\mu) - 12\,h^2\,l^2\,\mu^2\,\Phi + 6\,h\,l\,\mu\,\Phi^2 - \Phi^3\right)\}
\end{aligned}$$

$$\begin{aligned}
\beta_o \;=\; & \frac{l}{2\sqrt{15}} \times \{\; 1280\,h^8\,l^8\,\mu^6\;\left(-1-\mu+5\,\mu^2\right) \\
& - 640\,h^7\,l^7\,\mu^5\;\left(-9+7\,\mu+13\,\mu^2+13\,\mu^3\right)\;\Phi \\
& + 640\,h^6\,l^6\,\mu^4\;\left(-9+3\,\mu+25\,\mu^2+19\,\mu^3+4\,\mu^4\right)\;\Phi^2 \\
& - 320\,h^5\,l^5\,\mu^3\;\left(-9-\mu+41\,\mu^2+41\,\mu^3+12\,\mu^4\right)\;\Phi^3 \\
& + 80\,h^4\,l^4\,\mu^2\;\left(-9-5\,\mu+77\,\mu^2+107\,\mu^3+40\,\mu^4\right)\;\Phi^4 \\
& - 8\,h^3\,l^3\,\mu\;\left(-9-9\,\mu+245\,\mu^2+413\,\mu^3+200\,\mu^4\right)\;\Phi^5 \\
& + 112\,h^2\,l^2\,\mu^2\;\left(4+7\,\mu+4\,\mu^2\right)\;\Phi^6 \\
& - 16\,h\,l\,\mu\;\left(4+7\,\mu+4\,\mu^2\right)\;\Phi^7 + \left(4+7\,\mu+4\,\mu^2\right)\;\Phi^8\;\}^{\frac{1}{2}} \\
/ \quad & \{\;256\,h^8\,l^8\,(1-2\,\mu)^2\,\mu^4 - 640\,h^7\,l^7\,\mu^5\;(-1+2\,\mu)\;\Phi \\
& + 128\,h^6\,l^6\,\mu^4\;\left(-2+4\,\mu+3\,\mu^2\right)\;\Phi^2 - 32\,h^5\,l^5\,\mu^3\;\left(-2+4\,\mu+9\,\mu^2\right)\;\Phi^3 \\
& + 32\,h^4\,l^4\,\mu^2\;\left(-1+2\,\mu+4\,\mu^2\right)\;\Phi^4 - 8\,h^3\,l^3\,\mu\;\left(-1+2\,\mu+10\,\mu^2\right)\;\Phi^5 \\
& + 40\,h^2\,l^2\,\mu^2\,\Phi^6 - 10\,h\,l\,\mu\,\Phi^7 + \Phi^8\;\}^{\frac{1}{2}}
\end{aligned}$$

$$\begin{aligned}
\eta_o \;=\; & l\;\{\;16\,h^4\,l^4\;(-1+\mu)\;\mu^3 - 8\,h^3\,l^3\;(-4+\mu)\;\mu^3\,\Phi - 12\,h^2\,l^2\,\mu^2\;(2+\mu)\;\Phi^2 \\
& + 2\,h\,l\,\mu\;(4+5\,\mu)\;\Phi^3 - (1+2\,\mu)\;\Phi^4\;\} \\
/ \quad & 12\;\left\{8\,h^3\,l^3\,\mu\;(-1+2\,\mu) - 12\,h^2\,l^2\,\mu^2\,\Phi + 6\,h\,l\,\mu\,\Phi^2 - \Phi^3\right\}
\end{aligned}$$